Audiences in argumentation frameworks

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Abstract

Although reasoning about what is the case has been the historic focus of logic, reasoning about what should be done is an equally important capacity for an intelligent agent. Reasoning about what to do in a given situation - termed *practical reasoning* in the philosophical literature - has important differences from reasoning about what is the case. The acceptability of an argument for an action turns not only on what is true in the situation, but also on the values and aspirations of the agent to whom the argument is directed. There are three distinctive features of practical reasoning: first, that practical reasoning is situated in a context, directed towards a particular agent at a particular time; second, that since agents differ in their aspirations there is no right answer for all agents, and rational disagreement is always possible; third, that since no agent can specify the relative priority of its aspirations outside of a particular context, such prioritisation must be a product of practical reasoning and cannot be used as an input to it. In this paper we present a framework for practical reasoning which accommodates these three distinctive features. We use the notion of argumentation frameworks to capture the first feature. An extended form of argumentation framework in which values and aspirations can be represented is used to allow divergent opinions for different audiences, and complexity results relating to the extended framework are presented. We address the third feature using a formal description of a dialogue from which preferences over values emerge. Soundness and completeness results for these dialogues are given.

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1 Introduction

Reasoning about what should be done in a particular situation - termed *practical reasoning* in the philosophical literature - is carried out through a process of argumentation. Argumentation is essential because no completely compelling answer can be given: whereas in matters of belief, we at least should be constrained by what is actually the case, in matters of action no such constraints apply - we can choose what we will attempt to make the case. Even a norm as universal and deep seated as *thou shalt not kill* is acknowledged to permit of exceptions in circumstances of self defence and war. Thus whether arguments justifying or urging a course of action are acceptable will depend on the aspirations and *values* of the agent to which they are addressed: the *audience* for the argument. The importance of the audience for arguments was recognised and advocated by Perelman [20].

Arguments in practical reasoning provide presumptive reasons for performing an action. These presumptive arguments are then subject to a process of challenge, called *critical questioning* in [23]. These critical questions may take the form of other arguments, which can in turn be challenged, or may be answered by further arguments, resulting in a set of arguments constructed as the debate develops. An extension of Walton's account of practical reasoning is given in [2], which proposes an elaborated argument scheme for practical reasoning, which incorporates the value promoted by acceptance of the argument, and identifies all the ways in which it can be challenged. Although most of our discussion will treat arguments at a very abstract level, where we have need of a more particular structure for arguments, we will have this account in mind.

In this paper we will propose and explore a framework for the representation and evaluation of arguments in practical reasoning. Any such framework must account for some important phenomena associated with such reasoning. We will review these features in this section, and will structure the development of our framework in the remainder of this paper around them.

First, as is clear from the brief sketch of practical reasoning above, arguments cannot be considered in isolation. Whether an argument is acceptable or not depends on whether it can withstand or counter the other arguments put forward in the debate. Once the relevant arguments have been identified, whether a given argument is acceptable will depend on its belonging to a coherent subset of the arguments put forward which is able to defend itself against all attackers. We will call such a coherent subset a *position*. This notion of the acceptability of an argument deriving from membership of a defensible position has been explored in AI through the use of *argumentation frameworks* [12,4], and our account will be based on a framework of this sort. Dung's framework [12] will be recapitulated in section 2, and then extended as the paper proceeds. The reasoning involved in constructing argumentation frameworks and identifying positions within them is naturally modelled as a dialogue between a proponent and a critic. Dialogues for this purpose have been proposed in [8], [15] and [3], and we will make use of the way of exploring argument frameworks. Dialogues are discussed in section 5.

A second important feature of practical reasoning is that rational disagreement is possible, the acceptability of an argument depending in part on the audience to which it is addressed. Within Dung's framework it is possible for disagreement to be *represented* since argumentation frameworks may contain multiple incompatible defensible positions. The abstract nature of arguments, however, means that there is no information that can be used to *motivate* the choice of one option over another. Searle states the need to recognise that disagreement in practical reasoning cannot be eliminated as follows [21]:

> Assume universally valid and accepted standards of rationality, assume perfectly rational agents operating with perfect information, and you will find that rational disagreement will still occur; because, for example, the rational agents are likely to have different and inconsistent values and interests, each of which may be rationally acceptable.

What distinguishes different audiences are their values and interests, and in order to relate the positions acceptable to a given audience to the values and interests of that audience we need a way of relating arguments to such values and interests. Hunter [18] makes a proposal in terms of what he calls *resonance*, but we will build on *Value Based Argumentation Frameworks* (VAFs) proposed in [4], in which every argument is explicitly associated with a value promoted by its acceptance, and audiences are characterised by the relative ranking they give to these values. We will describe VAFs in section 3, their properties in section 4, and discuss the relationship between our proposal and Hunter's in section 7.

The above machinery can allow us to explain disagreement in terms of differences in the rankings of values between different audiences, but it does not allow us to explain these rankings. This brings us to the third feature of practical reasoning for which we wish to account - that we cannot assume that the parties to a debate will come with a clear ranking of values: rather these rankings appear to emerge during the course of the debate. We may quote Searle again:

> This answer [that we can rank values in advance] while acceptable as far as it goes [as an *ex post* explanation], mistakenly implies that the preferences are given *prior* to practical reasoning, whereas, it seems to me, they are typically the product of practical reasoning. And since ordered preferences are typically products of practical reason, they cannot be treated as its universal presupposition. [21]

The question of how value orders emerge during debate is explored in sections 6, in which we define a dialogue process for evaluating the status of arguments in a VAF, and in which we show how this process can be used to construct positions. In the course of constructing a position, the ordering of values will be determined.

Although it is not reasonable to assume that participants in a debate come with a firm value order, and so we wish to account for the emergence of such an order, neither do participants usually come to an debate with a completely open mind. Usually there will be some actions they are predisposed to perform, and others which they are reluctant to perform, and they will have a tendency to prefer arguments which match these predispositions. For example a politician forming a political programme may recognise that raising taxation is electorally inexpedient and so must exclude any arguments with the conclusion that taxes should be raised from the manifesto, while ensuring that arguments justifying actions bringing about core objectives are present: other arguments are acceptable in so far as they enable this. This kind of initial intuitive response to arguments will be used to drive the construction of positions and formation of a value order. A similar technique for constructing positions on the basis of Dung's framework has been proposed in [6]. Because this treatment does not make use of values, however, it cannot use these reasons for action to motivate choices, and there is no relation between the arguments which can be exploited to demand that choices are made in a consistent and coherent manner. Our extensions to include values enable us to impose this requirement of *moral consistency* on the reasoners.

Our overall aim is to provide a framework for modelling practical reasoning, which is based on sets of arguments together with information as to the other arguments they attack, and the values promoted by their acceptance. Our framework will account for three key features of practical reasoning: that evaluation of arguments is always in the context of a debate; that there is always potential for disagreement, explicable in terms of the different interests and values of the audiences; and that values are ordered in the course of the debate.

2 Dung's Argumentation Frameworks

We recall the following basic concepts that were introduced in Dung [12].

Definition 1 *An* argument system *is a pair* $\mathcal{H} = \langle \mathcal{X}, \mathcal{A} \rangle$*, in which* $\mathcal{X}$ *is a finite set of* arguments *and* $\mathcal{A} \subset \mathcal{X} \times \mathcal{X}$ *is the* attack relationship *for* $\mathcal{H}$*. A pair* $\langle x, y \rangle \in \mathcal{A}$ *is referred to as '*y is attacked by x*' or '*x attacks y*'. For* R*,* S *subsets of arguments in the system* $\mathcal{H}(\langle \mathcal{X}, \mathcal{A} \rangle)$*, we say that*

a. $s \in S$ *is* attacked *by* R *if there is some* $r \in R$ *such that* $\langle r, s \rangle \in \mathcal{A}$*.*
b. $x \in \mathcal{X}$ *is* acceptable with respect to S *if for every* $y \in \mathcal{X}$ *that attacks* x *there is some* $z \in S$ *that attacks* y*.*
c. S *is* conflict-free *if no argument in* S *is attacked by any other argument in* S*.*
d. A conflict-free set S *is* admissible *if every argument in* S *is acceptable with respect to* S*.*
e. S *is a* preferred extension *if it is a maximal (with respect to* $\subseteq$*) admissible set.*
f. S *is a* stable extension *if* S *is conflict free and every argument* $y \notin S$ *is attacked by* S*.*
g. $\mathcal{H}$ *is* coherent *if every preferred extension in* $\mathcal{H}$ *is also a stable extension.*

An argument x *is* credulously accepted *if there is* some *preferred extension containing it;* x *is* sceptically accepted *if it is a member of* every *preferred extension.*

The concepts of credulous and sceptical acceptance motivate the following decision problems that have been considered in [10,14].

Credulous Acceptance (CA)
Instance: Argument System, $\mathcal{H} = \langle \mathcal{X}, \mathcal{A} \rangle$, $x \in \mathcal{X}$.
Question: Is x credulously accepted in $\mathcal{H}$?

Sceptical Acceptance (SA)
Instance: Argument System, $\mathcal{H} = \langle \mathcal{X}, \mathcal{A} \rangle$, $x \in \mathcal{X}$.
Question: Is x sceptically accepted in $\mathcal{H}$?

The results of [10] establish that CA is NP–complete, while [14] have proven SA to be Π_2^p–complete [1]. Abstracting away concerns regarding the internal structure and representation of arguments affords a formalism which focuses on the relationship between individual arguments as a means of defining several different notions of acceptability. In this paper preferred extensions are of particular interest as these represent maximal coherent positions that can be defended against all attackers.

3 Value Based Argument Frameworks

Value Based Argument Frameworks (VAFs), were introduced in [3,4] as a mechanism with which to provide an interpretation of multiple preferred extensions in a single argument system. Thus, the basic formalism of Dung's framework as captured in Defn. 1 is extended to provide a semantics for distinguishing and choosing between consistent but incompatible belief sets through the use of *argument values*: arguments are seen as grounded on one of a finite number of abstract values and, where there are multiple preferred extensions, the interpretation of which to "accept" is treated in terms of preference orderings of the underlying value set according to the views held by a particular *audience*. Thus while in the standard Argumentation system the choice between preferred extensions is arbitrary, in a VAF we are able to motivate such choices by reference to the value ordering of the audience. The formal definition of such *value-based argumentation frameworks* is given below.

Definition 2 *A* value-based argumentation framework *(*VAF*), is defined by a triple $\langle \mathcal{H}(\mathcal{X}, \mathcal{A}), \mathcal{V}, \eta \rangle$, where $\mathcal{H}(\mathcal{X}, \mathcal{A})$ is an argument system, $\mathcal{V} = \{v_1, v_2, \ldots, v_k\}$ a set of k* values*, and $\eta : \mathcal{X} \to \mathcal{V}$ a mapping that associates a value $\eta(x) \in \mathcal{V}$ with each argument $x \in \mathcal{X}$.*

[1] Assuming that the argument system is *coherent*, SA is CO-NP–complete. While the problem of testing coherence is itself shown to Π_2^p–complete in [14], there are polynomial-time verifiable properties which ensure coherence, e.g. if the directed graph structure defined by $\langle \mathcal{X}, \mathcal{A} \rangle$ contains no odd-length cycles.

Central to the development presented in [4,3] and to the main themes of the present article is the concept of an *audience* [2]. The definition presented below refines the original form presented in [4].

Definition 3 *An* audience *for a* VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$, *is a binary relation* $\mathcal{R} \subset \mathcal{V} \times \mathcal{V}$ *whose (irreflexive) transitive closure,* $\mathcal{R}^*$, *is asymmetric, i.e. at most one of* $\langle v, v' \rangle$, $\langle v', v \rangle$ *are members of* $\mathcal{R}^*$ *for any distinct* v, $v' \in \mathcal{V}$. *We say that* v_i is preferred to v_j in the audience $\mathcal{R}$, *denoted* $v_i \succ_{\mathcal{R}} v_j$, *if* $\langle v_i, v_j \rangle \in \mathcal{R}^*$.

Viewing $\mathcal{R}^*$ in graph-theoretic terms, if $\mathcal{R}$ is an audience then $\mathcal{R}^*$ induces an acyclic directed graph over the vertex set $\mathcal{V}$. Unless otherwise stated, we identify audiences $\mathcal{R}$ with their closure $\mathcal{R}^*$, e.g. for $\mathcal{R} \subset \mathcal{V} \times \mathcal{V}$ given as part of an instance to some problem involving VAFs, we assume $\mathcal{R} = \mathcal{R}^*$.

Typically, an audience, $\mathcal{R}$, will not describe a *unique* total ordering of $\mathcal{V}$, but will be "compatible" with several distinct such orderings, i.e. all total orders, σ, for which $v_i \succ_\sigma v_j$ implies that $\langle v_j, v_i \rangle \notin \mathcal{R}$, i.e. if σ is a total ordering of $\mathcal{V}$ in which v_i is preferred to v_j then σ is compatible with the audience $\mathcal{R}$ only if v_j is *not* preferred to v_i in the audience $\mathcal{R}$. Formally, this set of compatible total orderings corresponds to the set of *linear extensions* of the (strict) partial order induced by $\mathcal{R}^*$.

Definition 4 *Let* $\mathcal{R}$ *be an audience,* α *is a* specific audience *(compatible with* $\mathcal{R}$*) if* α *is a* total ordering *of* $\mathcal{V}$ and

$$\forall\ v,\ v' \ \in\ \mathcal{V}\ \ \langle v, v' \rangle \in \alpha \Rightarrow \langle v', v \rangle \notin \mathcal{R}^*$$

We use $\chi(\mathcal{R})$ *to denote the set of specific audiences compatible with* $\mathcal{R}$.

Example 1 *For* $\mathcal{V} = \{A, B, C\}$.

1. *If* $\mathcal{R} = \emptyset$ *then* $\mathcal{R}^* = \emptyset$ *and*

$$\chi(\mathcal{R}) = \left\{ \begin{array}{lcl} \{\langle A,B\rangle,\langle B,C\rangle,\langle A,C\rangle\} & ; & \{\langle A,B\rangle,\langle C,B\rangle,\langle A,C\rangle\} \\ \{\langle B,A\rangle,\langle B,C\rangle,\langle A,C\rangle\} & ; & \{\langle B,A\rangle,\langle B,C\rangle,\langle C,A\rangle\} \\ \{\langle A,B\rangle,\langle C,B\rangle,\langle C,A\rangle\} & ; & \{\langle B,A\rangle,\langle C,B\rangle,\langle C,A\rangle\} \end{array} \right\}$$

[2] The term "audience", the use of which derives from [20] is also used in Hunter [18], although he distinguishes between audiences only in terms of beliefs, whereas [4] distinguishes them in terms of values, while also accommodating differences in beliefs.

Respectively corresponding to the total orderings,

$$A \succ_\sigma B \succ_\sigma C \;\; ; \;\; A \succ_\sigma C \succ_\sigma B$$
$$B \succ_\sigma A \succ_\sigma C \;\; ; \;\; B \succ_\sigma C \succ_\sigma A$$
$$C \succ_\sigma A \succ_\sigma B \;\; ; \;\; C \succ_\sigma B \succ_\sigma A$$

so that $\chi(\emptyset)$ *contains* every possible *specific audience. Reflecting this property, we refer to the special case* $\mathcal{R} = \emptyset$ *as the* universal audience. *In addition when the term specific audience is used without explicit reference to some audience* $\mathcal{R}$*, the underlying audience will be understood to be the universal audience.*

2. *If* $\mathcal{R} = \{\langle A,B\rangle, \langle B,C\rangle\}$ *then* $\mathcal{R}^* = \{\langle A,B\rangle, \langle B,C\rangle, \langle A,C\rangle\}$ *so that* $\chi(\mathcal{R}) = \{\mathcal{R}^*\}$*, i.e.* $\chi(\mathcal{R})$ *contains* exactly one *specific audience: that corresponding to the ordering* $A \succ_\sigma B \succ_\sigma C$*.*
3. *If* $\mathcal{R} = \{\langle A,B\rangle, \langle C,B\rangle\}$ *then* $\mathcal{R}^* = \mathcal{R}$ *and*

$$\chi(\mathcal{R}) \;\; = \;\; \{\{\langle A,B\rangle, \langle C,B\rangle, \langle A,C\rangle\} \; ; \; \{\langle A,B\rangle, \langle C,B\rangle, \langle C,A\rangle\}\}$$

corresponding to the orderings $A \succ_\sigma C \succ_\sigma B$ *and* $C \succ_\sigma A \succ_\sigma B$*.*

We adopt the convention of using lower case Greek letters – α, β, γ, etc. – when referring to *specific* audiences, whilst reserving upper case calligraphic symbols – $\mathcal{R}$, $\mathcal{S}$, $\mathcal{T}$, etc. – for audiences in the sense of Defn. 3.

Using VAFs, ideas analogous to those of admissible argument in standard argument systems are defined in the following way. Note that all these notions are now relative to some audience.

Definition 5 *Let* $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ *be a* VAF *and* $\mathcal{R}$ *an audience.*

a. *For arguments* x*,* y *in* $\mathcal{X}$*,* x *is a* successful attack *on* y *(or* x defeats y*)* with respect to the audience $\mathcal{R}$ *if:* $\langle x,y\rangle \in \mathcal{A}$ and *it is* not *the case that* $\eta(y) \succ_{\mathcal{R}} \eta(x)$*.*
b. *An argument* x *is* acceptable to the subset S *with respect to an audience* $\mathcal{R}$ *if: for every* $y \in \mathcal{X}$ *that* successfully attacks x *with respect to* $\mathcal{R}$*, there is some* $z \in S$ *that successfully attacks* y *with respect to* $\mathcal{R}$*.*
c. *A subset* S *of* $\mathcal{X}$ *is* conflict-free with respect to the audience $\mathcal{R}$ *if: for each* $\langle x,y\rangle \in S \times S$*, either* $\langle x,y\rangle \notin \mathcal{A}$ *or* $\eta(y) \succ_{\mathcal{R}} \eta(x)$*.*
d. *A subset* S *of* $\mathcal{X}$ *is* admissible *with respect to the audience* $\mathcal{R}$ *if:* S *is conflict free with respect to* $\mathcal{R}$ *and every* $x \in S$ *is acceptable to* S *with respect to* $\mathcal{R}$*.*
e. *A subset* S *is a* preferred extension *for the audience* $\mathcal{R}$ *if it is a maximal admissible set with respect to* $\mathcal{R}$*.*
f. *A subset* S *is a* stable extension *for the audience* $\mathcal{R}$ *if* S *is admissible with respect to* $\mathcal{R}$ *and for all* $y \notin S$ *there is some* $x \in S$ *which successfully*

attacks y with respect to $\mathcal{R}$.

We observe that in the case of $\mathcal{R}$ being the universal audience the forms described within Defn. 5 (a)–(f) match exactly the corresponding structures in Dung's framework as described in Defn. 1 (a)–(f).

A standard consistency requirement which we assume of the VAFs considered is that every directed cycle of arguments in these contains *at least two* differently valued arguments. We do not believe that this condition is overly restricting, since the existence of such cycles in VAFs can be seen as indicating a flaw in the formulation of the framework. While in standard argumentation frameworks cycles arise naturally, especially if we are dealing with uncertain or incomplete information, in VAFs odd length cycles in a single value represent paradoxes and even length cycles in a single value can be reduced to a dilemma which must be resolved by choosing one of the alternatives. Given the absence of cycles in a single value the following important property of VAFs with respect to specific audiences was demonstrated in [4].

Fact 6 *For every* specific *audience, α, $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ has a unique non-empty preferred extension, $P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$ which can be constructed by an algorithm that takes $O(|\mathcal{X}| + |\mathcal{A}|)$ steps. Furthermore $P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$ is a stable extension with respect to α.*

From Fact 6 it follows that, when attention is focused on one specific audience, analogues of many decision questions known to be intractable in the standard setting become significantly easier.

There are, however, a number of new issues that arise in the value-based framework from the fact that the relative ordering of different values promoted by distinct specific audiences results in arguments falling into one of three categories.

- C1. Arguments, x, that are in the preferred extension $P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$ for some specific audiences compatible with $\mathcal{R}$ but not all. Such arguments being called *subjectively acceptable* with respect to $\mathcal{R}$.
- C2. Arguments, x, that are in the preferred extension $P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$ for *every* specific audience compatible with $\mathcal{R}$. Such arguments being called *objectively acceptable* with respect to $\mathcal{R}$.
- C3. Arguments, x, that are not in any preferred extension $P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$ no matter which specific audience, α, compatible with $\mathcal{R}$ is used. Such arguments being called *indefensible* with respect to $\mathcal{R}$.

While we have introduced these in terms of arbitrary audiences, $\mathcal{R}$, the ideas presented in (C1)–(C3) are particularly of interest in the case of the universal audience: subjective acceptability indicating that there is *at least one* specific audience (total ordering of values) under which p is accepted; objective acceptability that p must be accepted irrespective of the value ordering described by a specific audience; and, in contrast, p being indefensible indicating that no specific audience can ever accept p.

As we indicated in the introductory discussion, one may often find in practical reasoning contexts that participants disagree on value priorities yet nonetheless have a "common stance" regarding the acceptability or otherwise of particular arguments. We may model such behaviours in terms of the following formalism.

Definition 7 *Let $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* VAF *and $\mathcal{R}$, $\mathcal{S}$ audiences. Given $x \in \mathcal{X}$ we say that $\mathcal{R}$ and $\mathcal{S}$ have* grounds for agreement over *x if either*

(1) There are specific audiences $\alpha \in \chi(\mathcal{R})$, $\beta \in \chi(\mathcal{S})$ such that

$$x \in P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha) \text{ and } x \in P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \beta)$$

or

(2) For all specific audiences $\alpha \in \chi(\mathcal{R}) \cup \chi(\mathcal{S})$, $x \notin P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$.

We say that $\mathcal{R}$ and $\mathcal{S}$ are at issue *over x if $\mathcal{R}$ and $\mathcal{S}$ do not have grounds for agreement over x, e.g. $x \in P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$ for some $\alpha \in \chi(\mathcal{R})$, but $x \notin P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \beta)$ for all $\beta \in \chi(\mathcal{S})$.*

We observe, in passing, that while it may seem more natural to define $\mathcal{R}$ and $\mathcal{S}$ as having "grounds for agreement over x" via the existence of some $\alpha \in \chi(\mathcal{R}) \cap \chi(\mathcal{S})$ for which $x \in P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$, such a choice turns out to be rather too restrictive: there could be no specific audience compatible with both $\mathcal{R}$ and $\mathcal{S}$, e.g. if $v \succ_{\mathcal{R}} v'$ and $v' \succ_{\mathcal{S}} v$, however, this need not not prevent x being subjectively acceptable with respect to both, i.e. the disagreement of the relative ordering of $\{v, v'\}$ is irrelevant to either audience's view of x.

In the following section we consider the computational complexity of some naturally arising decision questions regarding VAFs, audiences, and these classes of acceptability.

4 Audience Related Properties of VAFs

4.1 Complexity results

In this section [3] we consider the following decision problems:

[3] The results presented in this section are an extended and revised treatment of work originally described in [16,17].

Subjective Acceptance (SBA)
Instance: A VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$; argument $x \in \mathcal{X}$; audience $\mathcal{R}$.
Question: Is there a specific audience, $\alpha \in \chi(\mathcal{R})$ for which $x \in P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$?

Objective Acceptance (OBA)
Instance: A VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$; argument $x \in \mathcal{X}$; audience $\mathcal{R}$.
Question: Is $x \in P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$ for *every* specific audience $\alpha \in \chi(\mathcal{R})$?

Audiences at Issue (AAI)
Instance: A VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$; argument $x \in \mathcal{X}$; audiences $\mathcal{R}$, $\mathcal{S}$.
Question: Is there a specific audience $\alpha \in \chi(\mathcal{R})$ for which $x \in P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$ but *no* specific audience $\beta \in \chi(\mathcal{S})$ with $x \in P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \beta)$?

Our results below establish that

(1) SBA is NP–complete, even if $\mathcal{R}$ is the universal audience. (Theorem 8)
(2) OBA is CO-NP–complete, again even if $\mathcal{R}$ is the universal audience. (Theorem 9).
(3) AAI is D^p–complete, even in the special case $\mathcal{R} = \{\langle v, v' \rangle\}$ $\mathcal{S} = \{\langle v', v \rangle\}$ for distinct values v, $v' \in \mathcal{V}$. (Theorem 10)

We recall that D^p is the class of languages that may be expressed as the intersection of some language $L_1 \in \text{NP}$ with a language $L_2 \in \text{CO-NP}$.

Theorem 8 SBA *is* NP*–complete.*

Proof: For membership in NP simply non-deterministically choose an audience α from the $k!$ available then test if $\alpha \in \chi(\mathcal{R})$ and $x \in P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$, the latter test being accomplished by a polynomial-time algorithm, such as that given in [4].

We prove that SBA is NP–hard for the special case of $\mathcal{R}$ being the universal audience, using a reduction from 3-SAT. Given an instance $\Phi(Z_n) = \wedge_{i=1}^{m}(y_{i,1} \vee y_{i,2} \vee y_{i,3})$ of this we construct a VAF $\langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle$ and argument x such that $\langle \langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle, x, \emptyset \rangle$ is a positive instance of SBA if and only if $\Phi(Z_n)$ is satisfiable.

The framework uses $4n+m+1$ arguments which we denote $\{\Phi, C_1, \ldots, C_m\} \cup \cup_{i=1}^{n} \{p_i, q_i, r_i, s_i\}$. The relationship $\mathcal{A}_\Phi$ contains attacks $\langle C_j, \Phi \rangle$ for each $1 \leq j \leq m$ and attacks $\{\langle p_i, q_i \rangle, \langle q_i, r_i \rangle, \langle r_i, s_i \rangle, \langle s_i, p_i \rangle\}$ for each $1 \leq i \leq n$. The remaining attacks in $\mathcal{A}_\Phi$ are as follows. For each clause $y_{i,1} \vee y_{i,2} \vee y_{i,3}$ of $\Phi(Z_n)$ if $y_{i,j}$ is the literal z_k, the attack $\langle p_k, C_i \rangle$ is included in $\mathcal{A}_\Phi$; if $y_{i,j}$ is the literal $\neg z_k$, then the attack $\langle q_k, C_i \rangle$ is added.

The final part of the construction is to describe the value set $\mathcal{V}_\Phi$ and association of arguments with values prescribed by η. The set $\mathcal{V}_\Phi$ contains $2n + 1$ values

$\{con\} \cup \cup_{i=1}^{n} \{pos_i, neg_i\}$ and the mapping η assigns the value con to Φ and each argument in $\{C_1, \ldots, C_m\}$. Finally the arguments $\{p_i, r_i\}$ are mapped to the value pos_i and the arguments $\{q_i, s_i\}$ to the value neg_i. To complete the instance we set x to be Φ. We note that the constructed system satisfies the requirement that all cycles contain at least two distinct values.

Figure 1 illustrates the construction for the CNF.

$$\Phi(x, y, z) \quad = \quad (x \vee y \vee z)(\neg x \vee y \vee \neg z)(x \vee \neg y \vee z)$$

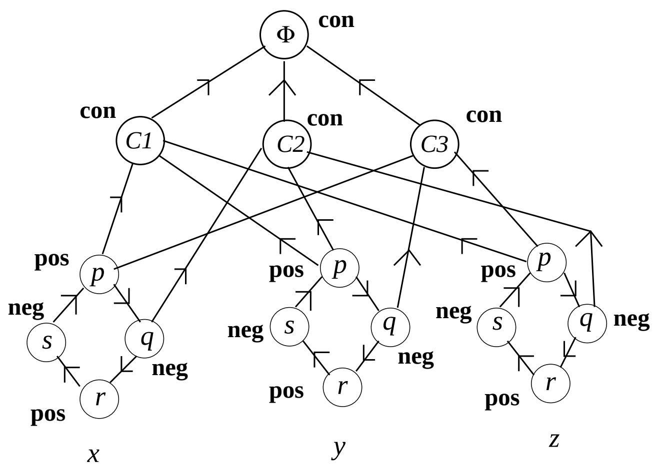


Fig. 1. VAF constructed for $\Phi(x, y, z) \quad = \quad (x \vee y \vee z)(\neg x \vee y \vee \neg z)(x \vee \neg y \vee z)$

We claim that $\langle\langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta\rangle, \Phi, \emptyset\rangle$ is a positive instance of SBA if and only if $\Phi(Z_n)$ is satisfiable.

Suppose first that $\Phi(Z_n)$ is satisfied by an instantiation $\langle a_1, a_2, \ldots, a_n\rangle$ of Z_n. Consider any specific audience α for which $pos_i \succ_\alpha neg_i$ if $a_i = \top$, $neg_i \succ_\alpha pos_i$ if $a_i = \bot$, and $v \succ_\alpha con$ for all $v \in \mathcal{V}_\Phi / \{con\}$. Since $\Phi(Z_n)$ is satisfied, for each C_i there is some literal $y_{i,j}$ that is assigned $\top$ in the instantiation $\langle a_1, \ldots, a_n\rangle$. Consider the arguments $\{p_k, q_k, r_k, s_k\}$ for which $y_{i,j} \in \{z_k, \neg z_k\}$. If $y_{i,j} = z_k$ then p_k is acceptable in $\{p_k, r_k\}$ and, in addition, p_k successfully attacks C_i with respect to α; if $y_{i,j} = \neg z_k$ then q_k is acceptable in $\{q_k, s_k\}$ and, again, successfully attacks C_i with respect to α. Thus every argument C_i is successfully attacked by an argument p_k or q_k and thence Φ together with these form an admissible set with respect to α. Thus we have a specific audience, α, with respect to which Φ is subjectively accepted.

On the other hand, suppose α is a specific audience for which Φ belongs to $P(\langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta\rangle, \alpha)$. It cannot be the case that $C_i \in P(\langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta\rangle, \alpha)$ since $\eta(\Phi) = \eta(C_i) = con$ and so the presence of any C_i would suffice to eliminate Φ. The specific audience α must therefore be such that every C_i is successfully attacked by one of its three possible attackers with respect to

α. Let $\langle t_1, t_2, \ldots, t_m \rangle$ be the choices which give these successful attacks on $\langle C_1, \ldots, C_m \rangle$. First observe that we cannot have $t_i = p_k$ and $t_j = q_k$ for any $1 \leq k \leq n$ and distinct C_i and C_j: under α either $\eta(p_k) \succ_\alpha \eta(q_k)$ and so q_k would not succeed in its attack or $\eta(q_k) \succ_\alpha \eta(p_k)$ with the attack by p_k failing. It follows that the instantiation of Z_n by $z_i = \top$ if $p_i \in \langle t_1, t_2, \ldots, t_m \rangle$, $z_i = \bot$ if $q_i \in \langle t_1, t_2, \ldots, t_m \rangle$ is well-defined and yields a true literal in every clause, i.e. results in a satisfying instantiation of $\Phi(Z_n)$. This completes the proof. □

Theorem 9 OBA *is* CO-NP*–complete.*

Proof: Membership in CO-NP follows by the algorithm which tests for all $k!$ specific audiences, α, whether $\alpha \in \chi(\mathcal{R}) \Rightarrow x \in P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$.

We again establish CO-NP–hardness for the special case of $\mathcal{R}$ being the universal audience, via a reduction from 3-UNSAT: the problem of deciding if a 3-CNF formula $\Phi(Z_n) = \wedge_{i=1}^{m} (y_{i,1} \vee y_{i,2} \vee y_{i,3})$ is unsatisfiable.

The reduction constructs an identical VAF to that of the previous theorem, but with one additional argument, $\{test\}$, having $\eta(test) = con$ and whose sole attacker is the argument Φ. Letting $\langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle'$ denote the resulting system, we claim that $\langle \langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle', test, \emptyset \rangle$ defines a positive instance of OBA if and only if Φ is unsatisfiable. From the proof of Theorem 8, *test* will fail to be acceptable with respect to any specific audience α for which Φ is admissible. Such an audience exists if and only if $\Phi(Z_n)$ is satisfiable. We therefore deduce that $\langle \langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle', test, \emptyset \rangle$ is accepted as an instance of OBA if and only if $\Phi(Z_n)$ is unsatisfiable. □

Theorem 10 AAI *is* D^p*–complete.*

Proof: For membership in D^p, define the language L_1 to be

$$\{ \langle \langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \mathcal{R}, \mathcal{S}, x \rangle \ : \ \exists \alpha \in \chi(\mathcal{R}) \text{ such that } x \in P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha) \}$$

Similarly, define L_2 as

$$\{ \langle \langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \mathcal{R}, \mathcal{S}, x \rangle \ : \ \forall \alpha \in \chi(\mathcal{S}) \ x \notin P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha) \}$$

Then $\langle \langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \mathcal{R}, \mathcal{S}, x \rangle$ is accepted as an instance of AAI if and only if it belongs to the set $L_1 \cap L_2$. Since it is immediate that $L_1 \in$ NP and $L_2 \in$ CO-NP this suffices to give AAI $\in \text{D}^p$.

We prove that AAI is D^p–hard for the special case of $\mathcal{R} = \{ \langle v, v' \rangle \}$ and $\mathcal{S} = \{ \langle v', v \rangle \}$ for v, v' distinct values in $\mathcal{V}$.

We first show that the problem *Critical Variable* (CV) is D^p–hard: instances of this comprise a CNF formula $\Phi(Z_n)$ and a variable $z \in Z_n$ with instances accepted if there is a satisfying instantiation in which $z = \top$ but no satisfying instantiation in which $z = \bot$. To see that CV is D^p–hard we use a reduction from the D^p–complete problem SAT-UNSAT. Given an instance $\langle \Phi_1(Z_n), \Phi_2(Z_n) \rangle$ of this, the instance $\langle \Psi, z \rangle$ of CV is simply $\langle (\neg z \vee \Phi_1) \wedge (z \vee \Phi_2), z \rangle$ where z is a new variable. We note that for Φ in CNF, $z \vee \Phi$ translates to the CNF formula in which every clause C of Φ is replaced by the clause $z \vee C$. It is easy to see that $\langle (\neg z \vee \Phi_1) \wedge (z \vee \Phi_2), z \rangle$ is a positive instance of CV if and only if $\langle \Phi_1(Z_n), \Phi_2(Z_n) \rangle$ is a positive instance of SAT-UNSAT: if Φ_1 is satisfiable then $(\neg z \vee \Phi_1) \wedge (z \vee \Phi_2)$ has a satisfying instantiation with $z = \top$ since it reduces to Φ_1; if Φ_2 is unsatisfiable then there is no satisfying instantiation with $z = \bot$ since the formula now reduces to Φ_2, hence if $\langle \Phi_1, \Phi_2 \rangle$ accepted as an instance of SAT-UNSAT then $\langle (\neg z \vee \Phi_1) \wedge (z \vee \Phi_2), z \rangle$ is accepted as an instance of CV. Similarly, if $\langle (\neg z \vee \Phi_1) \wedge (z \vee \Phi_2), z \rangle$ is a positive instance of CV then $(\neg z \vee \Phi_1) \wedge (z \vee \Phi_2)$ is satisfiable when $z = \top$, i.e. Φ_1 is satisfiable, and $(\neg z \vee \Phi_1) \wedge (z \vee \Phi_2)$ is unsatisfiable when $z = \bot$, i.e. Φ_2 is unsatisfiable.

The proof that AAI is D^p–hard now follows easily, using the reduction of Theorem 8: given an instance $\langle \Phi(Z_n), z \rangle$ of CV form the VAF $\langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle$ described in the proof of Theorem 8 (where we note that this trivially extends to arbitrary CNF formulae). Set the audiences in the instance of AAI to be $\mathcal{R} = \{\langle pos_z, neg_z \rangle\}$ and $\mathcal{S} = \{\langle neg_z, pos_z \rangle\}$. Finally fix the argument x to be Φ. Consider the resulting instance $\langle \langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle, \mathcal{R}, \mathcal{S}, \Phi \rangle$. If it is a positive instance of AAI then there is a specific audience $\alpha \in \chi(\mathcal{R})$ for which $\Phi \in P(\langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle, \alpha)$: this specific audience must have $pos_z \succ_\alpha neg_z$ (since $\langle pos_z, neg_z \rangle \in \mathcal{R}$): it has already been seen that this indicates $\Phi(Z_n)$ has a satisfying instantiation with $z = \top$. Similarly, if $\langle \langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle, \mathcal{R}, \mathcal{S}, \Phi \rangle$ is a positive instance of AAI, then $\Phi \notin P(\langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle, \alpha)$ for any specific audience in $\chi(\mathcal{S})$, i.e. all specific audiences within which $neg_z \succ_\alpha pos_z$. From our earlier analysis, $\Phi(Z_n)$ has no satisfying instantiation with $z = \bot$.

On the other hand should $\langle \Phi(Z_n), z \rangle$ be a positive instance of CV then the argument of Theorem 8 yields a specific audience α with $pos_z \succ_\alpha neg_z$ i.e. $\alpha \in \chi(\mathcal{R})$ for which $\Phi \in P(\langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle, \alpha)$ from a satisfying instantiation of $\Phi(Z_n)$ with $z = \top$. Similarly, the unsatisfiability of $\Phi(Z_n)$ when $z = \bot$ indicates that no specific audience α having $neg_z \succ_\alpha pos_z$, i.e. those in $\chi(\mathcal{S})$, will result in $\Phi \in P(\langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle, \alpha)$. We deduce that $\langle \Phi(Z_n), z \rangle$ is a positive instance of CV if and only if $\langle \langle \mathcal{X}_\Phi, \mathcal{A}_\Phi, \mathcal{V}_\Phi, \eta \rangle, \mathcal{R}, \mathcal{S}, \Phi \rangle$ is a positive instance of AAI, thereby establishing that AAI is D^p–complete. □

We have now arrived at the position where we can detect efficiently the arguments acceptable to any specific audience, but cannot guarantee that we will be able to determine the status of an argument with respect to the universal audience. We now consider another problem relating to VAFs which does admit an efficient solution, namely finding an audience for whom a subset of arguments represents a preferred extension, if one exists.

We begin by giving a formal statement of our problem:

Set Acceptance (SAC)
Instance: A VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$; a *subset* S of $\mathcal{X}$.
Question: Is there an audience $\mathcal{R}$ such that $\forall \alpha \in \chi(\mathcal{R})$, $S = P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$?

In this section we address this problem and some related applications. On first inspection, it might appear that, given the status of SBA, this too would be an intractable problem. We will show, however, that this pessimistic view is ill-founded: the critical difference between the two problems is that subjective acceptance concerns the existence of a specific audience with respect to which a *single* given argument is accepted; whereas the current problem asks for an audience with respect to which a given *set* of arguments defines the totality of what that audience *is capable of accepting*.

Consider the following algorithm:

FIND AUDIENCE
Instance: VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$; $S \subseteq \mathcal{X}$.
Returns: Audience $\mathcal{R}$ such that $\forall \alpha \in \chi(\mathcal{R})$, $S = P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle, \alpha)$ or FAIL if no such audience exists.

1. $\mathcal{R} \ := \ \emptyset$;
2. **for each** $\langle x, y \rangle \in S \times S$:
2.1. **if** $\langle x, y \rangle \in \mathcal{A}$ **then**
   a. **if** $\eta(x) = \eta(y)$ **then** report FAIL **else**

$$\mathcal{R} \ := \ \mathcal{R} \cup \{\langle \eta(y), \eta(x) \rangle\}$$

3. $\mathcal{R} \ := \ \mathcal{R}^*$, i.e. replace $\mathcal{R}$ with its transitive closure.
4. **if** $\mathcal{R}$ is not an audience (i.e. contains $\langle v, v' \rangle$ and $\langle v, v' \rangle$ for some v and v') **then** report FAIL **else**
   5. **for each** $z \notin S$
      a. **if** $\eta(z) = \eta(x)$ for some $x \in S$ **then**
         Find some $y \in S$ for which $\langle y, z \rangle \in \mathcal{A}$ *and* $\langle \eta(z), \eta(y) \rangle \notin \mathcal{R}$.
         report FAIL if no suitable $y \in S$ is found.

b. **else** – $\eta(z)$ does not occur as the value of any $x \in S$
Choose any $y \in S$ withh $\langle y, z\rangle \in \mathcal{A}$;
$\mathcal{R} := \mathcal{R} \cup \{\langle \eta(y), \eta(z)\rangle\}$
report FAIL if no $y \in S$ attacks z.

6. $\mathcal{R} := \mathcal{R}^*$.
7. **return** $\mathcal{R}$

Theorem 11 *Given an instance* $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta\rangle$ *and* $S \subseteq \mathcal{X}$ *the algorithm* FIND AUDIENCE *returns an audience* $\mathcal{R}$ *such that* $\forall \alpha \in \chi(\mathcal{R})$, $S = P(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta\rangle, \alpha)$ *or reports* FAIL *if no such audience exists. Furthermore the time taken is* $O(|\mathcal{X}|^2)$.

Proof: Steps (2) and (3) of FIND AUDIENCE construct a partial ordering of the values in S that satisfies the requirement that S must be conflict-free with respect to the audience: thus each $\langle x, y\rangle \in \mathcal{A}$ for which both x and y are in S forces an ordering of the values $\{\eta(x), \eta(y)\}$ according to the constraints specified in Defn. 5(c). All constraints arising thus are added by the loop comprising (2), resulting in the set of constraints $\mathcal{R}$ upon completion. At step (3), this set is extended to include all of the additional pair-wise orderings arising through the property that if $\langle \eta(x), \eta(y)\rangle \in \mathcal{R}$ and $\langle \eta(y), \eta(z)\rangle \in \mathcal{R}$ then any audience consistent with $\mathcal{R}$ must have $\eta(x) \succ_{\mathcal{R}} \eta(z)$: constructing all of the pair-wise orderings that should be included simply involves computing the (irreflexive) transitive closure of the relations identified after (2) has completed. Step (4) deals with the requirement that since the audience relation must be asymmetric the set of pairs $\mathcal{R}$ cannot contain both $\langle v_i, v_j\rangle$ and $\langle v_j, v_i\rangle$: this would happen if, for example, there were $\{x, y, z\} \in S$ with $\langle x, y\rangle \in \mathcal{A}$, $\langle y, z\rangle \in \mathcal{A}$ and $\eta(x) = \eta(z)$. Since (3) has formed the transitive closure of the constraint relationship identified in (2), the "consistency" test in (4) involves checking that for each $x \in S$ the pair $\langle \eta(x), \eta(x)\rangle$ has not been added. Step (5) is concerned with checking that S is *maximal* with respect to the audience that has been constructed in the earlier stages. Again, from Definition 5, this simply involves testing for each argument $z \notin S$, that z cannot be added to S without creating a conflict. There are two possibilities. Firstly, the value $\eta(z)$ is among those considered in S: in this case it suffices to ensure that z is successfully attacked by some $y \in S$. Secondly, the value $\eta(z)$ is distinct from any value used in S: in this case it suffices to find any $y \in S$ that attacks z. □

4.3 Discussion: Algorithms and Complexity in Dung's Framework and VAF*s*

Turning to the relative complexity of seemingly related problems within VAFs and Dung's framework, it could appear that the result presented in Theorem 11 is at odds with that of Theorem 8. That this is not the case is easily seen by noting that should the algorithm analysed in Theorem 11 return FAIL given some set $S \subseteq \mathcal{X}$ then this does *not* imply that each argument in S is indefensible. For example, consider the extreme case where S contains a single argument x: each of the following is possible

(1) x is subjectively acceptable and FIND AUDIENCE reports FAIL.
(2) x is *objectively* acceptable and FIND AUDIENCE reports FAIL.
(3) $\{x\}$ is admissible w.r.t some $\mathcal{R}$ and FIND AUDIENCE reports FAIL.

The algorithm FIND AUDIENCE returns $\mathcal{R}$ (rather than FAIL) only if S is both *maximal* and *admissible*: thus, the first two cases will arise whenever x is attacked by another argument; the final case would occur whenever S was not maximal, i.e. there is some $z \notin S$ that is not successfully attacked by any member of S.

We note that changing the loop condition governing (5) in this algorithm to

for each $\langle z, y\rangle \in \mathcal{A}$ for which $z \notin S$ and $y \in S$

(with the remainder of (5) unaltered) gives an algorithm to construct an audience with respect to which S is an *admissible set*. It is well-known that checking if a given set of arguments is admissible or defines a *stable* extension (in the schema of [12]) can be done efficiently. In VAFs the unique preferred extension with respect to a specific audience is a stable extension, so we may interpret FIND AUDIENCE (and its modification) as confirming that testing if S is admissible or stable remains tractable within VAFs, despite the additional constraints arising from the concept of audience.

The concepts of subjective and objective acceptance have a (superficial) similarity to those of credulous and sceptical acceptance. In this light, coupled with the facts that deciding if an argument is credulously accepted is NP–complete [10], deciding if an argument is sceptically accepted is Π_2^p–complete [14] the intractability of SBA and OBA is unsurprising. [4]

We note, however, that there are a number of differences between the two groups of problems. One obvious distinction is in the form of the search-space structures: CA and SA ranging over subsets of $\mathcal{X}$; SBA and OBA over possible (total) orderings of $\mathcal{V}$. In addition, we have the following,

Theorem 12

a. SBA$(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta\rangle, x, \emptyset) \not\Rightarrow$ CA$(\langle \mathcal{X}, \mathcal{A}\rangle, x)$.
b. OBA$(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta\rangle, x, \emptyset) \not\Rightarrow$ SA$(\langle \mathcal{X}, \mathcal{A}\rangle, x)$.

[4] In addition one can note the structural similarity of the VAF constructed in the reduction from 3-SAT of Theorem 8 to the argument system constructed in reductions from 3-SAT to credulous acceptance, e.g. [15, Defn. 7, p. 234].

c. $\text{OBA}(\langle\mathcal{X},\mathcal{A},\mathcal{V},\eta\rangle,x,\emptyset)\not\Rightarrow\text{CA}(\langle\mathcal{X},\mathcal{A}\rangle,x)$.
d. $\text{SA}(\langle\mathcal{X},\mathcal{A}\rangle,x)\not\Rightarrow\text{OBA}(\langle\mathcal{X},\mathcal{A},\mathcal{V},\eta\rangle,x,\emptyset)$.
e. $\text{CA}(\langle\mathcal{X},\mathcal{A}\rangle,x)\not\Rightarrow\text{SBA}(\langle\mathcal{X},\mathcal{A},\mathcal{V},\eta\rangle,x,\emptyset)$.
f. $\text{SA}(\langle\mathcal{X},\mathcal{A}\rangle,x)\not\Rightarrow\text{SBA}(\langle\mathcal{X},\mathcal{A},\mathcal{V},\eta\rangle,x,\emptyset)$.

Proof: Consider the three VAFs within which $\mathcal{V}=\{A,B\}$ shown in Figure 2.

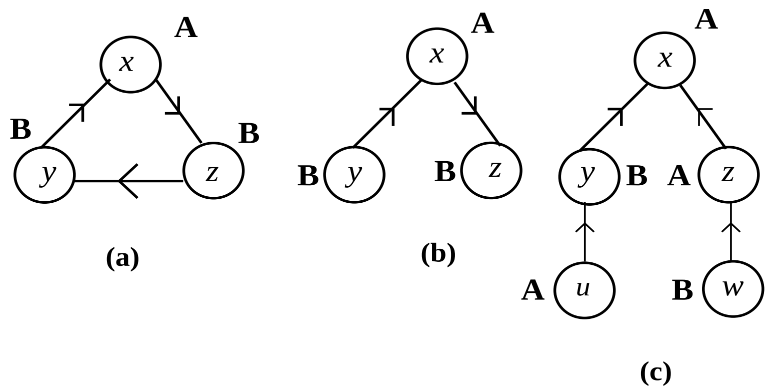


Fig. 2. Example VAFs in proof of Theorem 12

For the system of Figure 2(a), the unique preferred extension is the empty set. When interpreted as a VAF, however, the argument x is objectively acceptable, and the argument z is subjectively acceptable as witnessed by the specific audience $B\succ A$. These establish cases (a–c) of the theorem.

In the system of Figure 2(b), there is (again) a unique preferred extension comprising the arguments $\{y,z\}$ (thus both are sceptically accepted). For the specific audience $A\succ B$ the associated preferred extesnion is $\{x,z\}$: this does not contain y thus proving (d).

Finally noting that (f) subsumes (e), in the system depicted in Figure 2(c), the preferred extension is $\{u,w,x\}$. The argument x, however, is indefensible in the VAF interpretation: the specific audience $A\succ B$ leaves no counterattack to the attack by z on x since w does not succesfully attack z with respect to the audience $A\succ B$. Similarly, for the remaining specific audience $B\succ A$, the attack by y and x can no longer be countered since u does not successfully attack y with respect to the audience $B\succ A$. □

Thus, it is not generally possible to deduce acceptance by specific audiences or indefensibility from corresponding acceptance classes in the underlying argument system produced by abstracting away references to values.

5 Dialogue Processes for Determining Argument Status

Given a standard argument system – $\langle \mathcal{X}, \mathcal{A} \rangle$ – and an argument $x \in \mathcal{X}$ resolving the question of whether or not x is credulously accepted can be viewed, in a natural way, as a *dialogue* between a proponent of x (whom we shall denote PRO) and an opposing player (denoted as OPP): the latter advancing attacking arguments, y; the former selecting, in turn, arguments that counterattack these. This high-level abstraction of dialogue in argument systems is already proposed within Dung's original presentation [12, p. 332], but is not subsequently developed therein. Subsequently, however, a number of formalised dialogue processes building on standard argument systems have been developed and analysed. In this section we review such approaches and, informally, present the notion of a "position" within a VAF: positions and dialogue mechanisms for constructing these are treated in depth in Section 6.

An important generic formalism for defining dialogue schemes was introduced by Jakobovits and Vermeir [19]. The model defined presents *dialogue games* on argument systems – $\langle \mathcal{X}, \mathcal{A} \rangle$ – as a sequence of *moves*, $\mu_0\mu_1 \cdots \mu_r \cdots$ made by the players PRO and OPP. A specific instantiation of this generic scheme must provide a repertoire of *move types* (with particular move types involving parameters such as, e.g. individual arguments); and a *legal move* function that defines for any "partial" dialogue $\mu_0 \cdots \mu_r$ which of $\{\mathrm{PRO}, \mathrm{OPP}\}$ should contribute the next move, μ_{r+1} and the instantiations of available moves for the player concerned. While it is, usually, the case that PRO makes the initial move and that players alternate turns thereafter, it is convenient to relax this under certain conditions, e.g. when the case being set out by PRO involves advancing a number of separate arguments prior to the dialogue proper commencing: such a convention is adopted in the dialogue mechanisms presented in Section 6. The formalism of [19] has been used to specify dialogue procedures for credulous reasoning and determination of preferred sets in work of Cayrol *et al.*[7,8]. As has been shown in Amgoud and Cayrol [1] the formalism is general enough to accommodate models that develop Dung's frameworks: [1] describing instantiations yielding dialogue mechanisms in Preference-based argument frameworks.

In addition to these schemes, one dialogue process – the class of *Two Party Immediate response disputes* (TPI-disputes) – has been the subject of detailed analysis. This approach was introduced by Vreeswijk and Prakken [22], and can be interpreted as restricting the arguments that may be used by PRO and OPP to those that directly attack the most recent argument advanced: thus if x is the argument put forward by PRO in move μ_r then the argument y played by OPP in μ_{r+1} must be such that $\langle y, x \rangle \in \mathcal{A}$. The resulting dialectic proof procedures described in [22] are proven to be sound and complete for deciding *credulous* acceptance. In the case of proving *sceptical* acceptance, TPI-disputes are sound and complete when applied to *coherent* argument systems. The development of sound and complete proof mechanisms for sceptical argumentation raises a number of difficulties as discussed in [14, pp. 201-2]. An alternative formalisation of TPI-disputes is described and studied in [15]: building on the observation of [22, p. 247] that provision must be made for both PRO and OPP to "backtrack" to some earlier point in the dialogue, [15] analyse the "efficiency" of TPI-disputes when applied as a propositional proof theory, showing it to be polynomially-equivalent [5] to the CUT-free Gentzen calculus. Thus, there are examples in which demonstrations that an argument is *not* credulously accepted require exponentially many moves in the total number of arguments in the system. Finally, there is the recent work of Dung *et al.* [13], proposing a novel approach to the synthesis of dialectic proof procedures within the assumption-based framework of [5]: in this the use of "backward reasoning" is promoted as a means of resolving whether an argument is admissible.

In Section 6 our aim is to develop similar dialogue based mechanisms to those outlined above, but tailored to the characteristics of value-based argument frameworks. [6] We note here that starting from the basis of a VAF and the universal audience, player PRO has available options in addition to simply bringing forward "new" arguments to counterattack those proposed by OPP: PRO can also render an attack ineffective by expressing a suitable value ordering. In fact we wish to consider such dialogues as not so much concerned with "individual" arguments but rather as considering whether a set of arguments could be collectively acceptable to *some* audience. Thus in Section 6 our principal interest is in whether a subset $S \subseteq \mathcal{X}$ in a VAF defines a *position*: that is, whether there is some set T containing S and an audience $\mathcal{R}$ for which T is admissible.

6 Dialogue Processes for Position Construction in VAFs

6.1 Definition of a position

The notion of a position given at the end of the last section already addresses two of the crucial features for practical reasoning identified in the introduction: we are dealing with sets of arguments within an argumentation framework, and so considering the context, and the fact that different positions will be acceptable to different audiences captures the desired notion of rational dis-

[5] The concept of "polynomially-equivalence" of proof systems derives from Cook and Reckhow [9]

[6] A very brief overview of the ideas presented in this section was given in [11].

agreement. In this section we will address the remaining requirement: that the ordering of values should emerge from the debate, on the basis of some intuitive predisposition towards certain actions and reluctance to perform others.

First, to allow reasoners to have certain arguments they wish to include in a position, and others they wish to exclude, while they are indifferent to the rest, we extend the definition of a VAF as follows:

Definition 13 *A* VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ *is* DOR-partitioned *if* $\mathcal{X} = D \cup O \cup R$ *for disjoint sets* D, O *and* R*, which denote respectively a set of* desired *arguments, a set of* optional *arguments and a set of* rejected *arguments. We use* $\mathrm{Des}(\mathcal{X})$ *to denote* D*,* $\mathrm{Opt}(\mathcal{X})$ *to denote* O *and* $\mathrm{Rej}(\mathcal{X})$ *to denote* R*. A* DOR*-partitioned* VAF *is called a* DOR-VAF*.*

An admissible set which can be adopted as a position in a DOR-VAF, is a set that contains the desired arguments and possibly some optional arguments, whose role is to help a desired argument to be acceptable to the position, by "defending" it against its attackers. Formally:

Definition 14 *Given a* VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$*, an argument* y *is a* defender *of an argument* x *with respect to an audience* $\mathcal{R}$ *if and only if there is a finite sequence* $a_0, \ldots a_{2n}$ *such that* $x = a_0$*,* $y = a_{2n}$*, and* $\forall i, 0 \leq i \leq (2n-1)$*,* a_{i+1} *successfully attacks* a_i *with respect to* $\mathcal{R}$*.*

The new notion of a position is defined via:

Definition 15 *Given a* DOR-VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$*, a set* $S = \mathrm{Des}(\mathcal{X}) \cup Y$ *where* $Y \subseteq \mathrm{Opt}(\mathcal{X})$*, is a* position *if and only if there exists at least one audience* $\mathcal{R}$ *w.r.t. which* S *is admissible and* $\forall y \in Y, \exists x \in \mathrm{Des}(\mathcal{X})$ *such that* y *is a defender of* x*. An audience w.r.t. which* S *is a position is said to be a* corresponding audience *of* S*.*

This new notion of a position accomodates the third feature of practical reasoning: the preferences between values are not given as an input on the basis of which the position is constructed, but are a result of constructing the position.

6.2 Development of a position

6.2.1 General idea

In order to build a position, one may start by considering the set of desired arguments. This set must be first tested to demonstrate that there is at least one audience w.r.t. which it is conflict-free. It may be that this condition can only be satisfied by imposing some value preferences. If we can satisfy this test we must next ensure that any defeated argument in the set has a defender in the set w.r.t. at least one audience. To this end, some optional arguments may be added to the set as defenders of defeated arguments and/or some additional constraints on the ordering of values may be imposed. We would like such extensions of the position under development to be kept to a minimum. If the process succeeds, then the set developed is a position and the set of constraints determined by the construction can be extended into a corresponding audience of this position, by taking its transitive closure. Otherwise, the user has to reconsider the partition of the set of arguments; such issues are the subject of ongoing research.

This construction can be presented in the form of a *dialogue* between two players. One, the *opponent*, outlines why the set under development is not yet a position, by identifying arguments which defeat members of the set. The other, the *proponent*, tries to make the set under development a position by extending it with some optional arguments and/or some constraints between values. If the opponent has been left with no legal move available, then the set of arguments played by the proponent is a position and the set of constraints advanced can be extended into a corresponding audience. On the other hand, if the proponent has no legal move available then the set of desired arguments cannot be extended into a position.

This presentation in a dialogue form has the main advantage of making clear why some constraints between values must be imposed, and why some optional arguments must belong to the position. Moreover, it is highly appropriate to the dialectical nature of practical reasoning identified above.

In Section 6.2.2, we present a formal dialogue framework that we instantiate in Section 6.2.3 in order to check if a set of desired arguments is conflict-free for at least one audience. We instantiate the dialogue framework in Section 6.2.4 to check if a conflict-free set of desired arguments can be made acceptable. Finally, in Section 6.2.5 we combine these two instantiations of the dialogue framework to construct positions, and we give an example of such a construction.

6.2.2 Dialogue framework

A dialogue framework to prove the acceptability of arguments in Dung's argument system has been developed by [19] and refined in [8]. We extend this last framework to deal with the development of positions in a DOR-VAF.

Informally, a dialogue framework provides a definition of the players, the moves, the rules and conditions under which the dialogue terminates, i.e. those situations wherein the current player has no legal move in the dialogue. In order to capture the construction of positions, the dialogue framework we define comprises two players, PRO and OPP. The rules are expressed in a so-called 'legal-move function'. Regarding the definition of a move, since playing an argument may be possible only if some preferences between values hold, a move must comprise an argument and a set of value preferences. In particular, a player may propose some ordering of values, i.e without any specific argument being involved (for example, when he wants to make a set of desired arguments conflict-free for at least one audience). To this end, it is convenient to extend the set of arguments of a DOR-VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ with an 'empty argument' that we denote by $\_$. This argument can be used if the proponent's move is only to advance a value ordering. We denote by $\mathcal{X}^-$ the set $\mathcal{X} \cup \{\_\}$.

Definition 16 *Let* $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta \rangle$ *be a* DOR-VAF. *A* move *in* $\mathcal{X}^-$ *is a pair* $[P, \langle X, V \rangle]$ *where* $P \in \{\text{PRO}, \text{OPP}\}$, $X \in \mathcal{X}^-$, *and* $V \subseteq \mathcal{V} \times \mathcal{V}$. PRO *denotes the* proponent *and* OPP *denotes the* opponent*.*

For a move $\mu = [P, \langle X, V \rangle]$*, we use* $\text{pl}(\mu)$ *to denote* P*,* $\text{arg}(\mu)$ *to denote* X*, and* $\text{val}(\mu)$ *to denote* V*. The set of moves is denoted by* $\mathcal{M}$ *with* $\mathcal{M}^*$ *being the set of finite sequences of moves.*

Let $\phi : \mathcal{M}^* \to 2^{\mathcal{X}^- \times 2^{\mathcal{V} \times \mathcal{V}}}$ *be a* legal-move function*. A* dialogue *(or* ϕ*-dialogue)* d *about* $S = \{a_1, a_2, \ldots, a_n\} \subseteq \mathcal{X}$ *is a countable sequence* $\mu_{0_1}\mu_{0_2} \ldots \mu_{0_n}\mu_1\mu_2 \ldots$ *of moves in* $\mathcal{X}^-$ *such that the following conditions hold:*

(1) $\text{pl}(\mu_{0_k}) = \text{PRO}$*,* $\text{arg}(\mu_{0_k}) = a_k$*, and* $\text{val}(\mu_{0_k}) = \emptyset$ *for* $1 \leq k \leq n$
(2) $\text{pl}(\mu_1) = \text{OPP}$ *and* $\text{pl}(\mu_i) \neq \text{pl}(\mu_{i+1})$*, for* $i \geq 1$
(3) $\langle \text{arg}(\mu_{i+1}), \text{val}(\mu_{i+1}) \rangle \in \phi(\mu_{0_1}\mu_{0_2} \ldots \mu_{0_n}\mu_1 \ldots \mu_i)$*.*

In a dialogue about a set of arguments, the first n moves are played by PRO to put forward the elements of the set, without any constraint on the value of these arguments (*1*), and the subsequent moves are played alternately by OPP and PRO (*2*). The legal-move function defines at every step what moves can be used to continue the dialogue (*3*). We do not impose the requirement that $\text{arg}(\mu_{i+1})$ must attack $\text{arg}(\mu_i)$, because we want a dialogue to be sequential, so we need to let OPP try all possible answers to any of PRO's arguments, but only one at a time.

Let $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a DOR-VAF, $S \subseteq \mathcal{X}$ and $d = \mu_{0_1} \ldots \mu_{0_n}\mu_1\mu_2 \ldots \mu_i$ be a finite ϕ-dialogue about S. We denote μ_i by $\text{last}(d)$ and write $\phi(d)$ for $\phi(\mu_{0_1} \ldots \mu_{0_n}\mu_1\mu_2 \ldots \mu_i)$. In addition, $\text{argPRO}(d)$ (resp. $\text{valPRO}(d)$) will denote the set of arguments (resp. values) played by PRO in d.

Now that we have a way to define a dialogue and the rules of a dialogue, let us define when a dialogue terminates (i.e. cannot be continued).

Definition 17 *Let* $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta \rangle$ *be a DOR-*VAF*,* ϕ *be a legal-move function, and* d *be a finite* ϕ*-dialogue.* d cannot be continued *if* $\phi(d) = \emptyset$*.* d *is said to be* won by PRO *if and only if* d *cannot be continued, and* $\text{pl}(\text{last}(d)) = \text{PRO}$*.*

We introduce the notion of a definite attack and additional notations to instantiate the dialogue framework to develop positions.

Definition 18 *Let* $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ *be a* VAF*,* $\mathcal{R}$ *be an audience, and* x *and* y *be two arguments of* $\mathcal{X}$*.* x definitely attacks y *with respect to* $\mathcal{R}$ *if:* $\langle x, y \rangle \in \mathcal{A}$ *and,* $\eta(x) = \eta(y)$ *or* $\langle \eta(x), \eta(y) \rangle \in \mathcal{R}^*$*.*

Notice that, if x definitely attacks y, then x successfully attacks y. Now, given an audience $\mathcal{R}$ and $x \in \mathcal{X}^-$:

- $\mathcal{A}^+_\mathcal{R}(x) = \{y \in \mathcal{X} \mid x \text{ successfully attacks } y \text{ w.r.t. } \mathcal{R}\}$,
- $\mathcal{A}^{++}_\mathcal{R}(x) = \{y \in \mathcal{X} \mid x \text{ definitely attacks } y \text{ w.r.t. } \mathcal{R}\}$,
- $\mathcal{A}^-_\mathcal{R}(x) = \{y \in \mathcal{X} \mid y \text{ successfully attacks } x \text{ w.r.t. } \mathcal{R}\}$,
- $\mathcal{A}^{--}_\mathcal{R}(x) = \{y \in \mathcal{X} \mid y \text{ definitely attacks } x \text{ w.r.t. } \mathcal{R}\}$,
- $\mathcal{A}^\pm_\mathcal{R}(x) = \mathcal{A}^+_\mathcal{R}(x) \cup \mathcal{A}^-_\mathcal{R}(x)$.

Note that $\mathcal{A}^+_\mathcal{R}(\_) = \mathcal{A}^-_\mathcal{R}(\_) = \mathcal{A}^{--}_\mathcal{R}(\_) = \mathcal{A}^{++}_\mathcal{R}(\_) = \emptyset$. Moreover, given a set $S \subseteq \mathcal{X}$ and $\varepsilon \in \{+, -, \pm, ++, --\}$, $\mathcal{A}^\varepsilon_\mathcal{R}(S) = \bigcup_{x \in S} \mathcal{A}^\varepsilon_\mathcal{R}(x)$.

6.2.3 Checking conflict-freeness

Let $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a DOR-VAF and $\mathcal{R}$ be an audience. $\text{Des}(\mathcal{X})$ is not conflict-free w.r.t. $\mathcal{R}$ if there are two desired arguments x and y such that y successfully attacks x, that is, $\langle y, x \rangle \in \mathcal{A}$ and $\langle \eta(x), \eta(y) \rangle \notin \mathcal{R}$. In order to make $\text{Des}(\mathcal{X})$ conflict-free, the value of x should be made preferred to the value of y, that is, $\langle \eta(x), \eta(y) \rangle$ added to $\mathcal{R}$. This is possible only if $\mathcal{R} \cup \{\langle \eta(x), \eta(y) \rangle\}$ is an audience.

Consider a dialogue d about $\text{Des}(\mathcal{X})$, based on a legal-move function where OPP plays moves using arguments such as y and the value ordering is empty, and where PRO only exhibits constraints on the value of these arguments. Then the set of arguments played by PRO in d (i.e. $\text{argPRO}(d)$) is $\text{Des}(\mathcal{X})$, possibly along with $\{\_\}$. The value orderings played by PRO in d (i.e. $\text{valPRO}(d)$) must be the audience w.r.t. which moves are made. Formally:

Definition 19 *Let* $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta \rangle$ *be a* DOR-VAF*,* d *be a dialogue about* $\text{Des}(\mathcal{X})$ *and* $\mathcal{R} = \text{valPRO}(d)$*.* $\phi_1 : \mathcal{M}^* \to 2^{\mathcal{X}^- \times 2^{\mathcal{V} \times \mathcal{V}}}$ *is defined by:*

- *if the last move of* d *is by* PRO *(next move is by* OPP*),*

$$\phi_1(d) = \bigcup_{y \in \mathcal{A}^-_\mathcal{R}(\text{argPRO}(d)) \cap \text{argPRO}(d)} \{\langle y, \emptyset \rangle\};$$

- *if the last move of d is by* OPP *(next move is by* PRO*), let* $y = \text{arg}(\text{last}(d))$, $V = \bigcup_{x \in \mathcal{A}^+_{\mathcal{R}}(y) \cap \text{argPRO}(d)} \{\langle \eta(x), \eta(y) \rangle\}$,

$$\phi_1(d) = \begin{cases} \{\langle \_\,, V \rangle\} & \text{if } \mathcal{R} \cup V \text{ is an audience,} \\ \emptyset & \text{otherwise.} \end{cases}$$

The dialogue framework instantiated with the legal-move function ϕ_1, is sound and complete w.r.t. the determination of an audience w.r.t. which a set of arguments is conflict-free as shown by the two following properties.

Property 1 (Soundness of ϕ_1) *Let $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* VAF *and $S \subseteq \mathcal{X}$. If d is a ϕ_1-dialogue about S won by* PRO*, then S is conflict-free w.r.t. audience* valPRO(d).

Lemma 1 *Let $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* VAF *and $S \subseteq \mathcal{X}$. If d is a ϕ_1-dialogue about S, the last move of which is played by* PRO*, then* valPRO(d) *is an audience.*

Proof: Let $S = \{a_1, \ldots, a_n\} \subseteq \mathcal{X}$. Let d be a ϕ_1-dialogue about S, the last move of which is played by PRO. If the length of d is lower than or equal to n, then all the moves of d have the form $[\text{PRO}, \langle a_i, \emptyset \rangle]$ where a_i is an argument of S ($1 \leq i \leq n$). So $\text{valPRO}(d) = \emptyset$, and then valPRO($d$) is obviously an audience. If the length of d is strictly greater than n, then d has the form $d = d'.[\text{OPP}, \langle y, \emptyset \rangle].[\text{PRO}, \langle \_\,, V \rangle]$, where V is, by definition, a set of value preferences such that $\text{valPRO}(d'.[\text{OPP}, \langle y, \emptyset \rangle]) \cup V$ is an audience. Since $\text{valPRO}(d'.[\text{OPP}, \langle y, \emptyset \rangle]) \cup V = \text{valPRO}(d)$, valPRO($d$) is an audience. □

Lemma 2 *Let $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* VAF*. Let d be a ϕ_1-dialogue about a set $S \subseteq \mathcal{X}$.*

- *If d is of length strictly lower than $|S|$ then* $\text{argPRO}(d) \subseteq S$.
- *If d is of length $|S|$ or $|S| + 1$ then* $\text{argPRO}(d) = S$.
- *If d is of length strictly greater than $|S| + 1$ then* $\text{argPRO}(d) = S \cup \{\_\}$.

Proof: Obvious, since by definition, the first $|S|$ moves of d are by PRO and contain S's arguments; all the following moves by PRO contain the empty argument. □

Proof: *(of Property 1)* Let $S \subseteq \mathcal{X}$. Let d be a ϕ_1-dialogue about S won by PRO. Let $\mathcal{R} = \text{valPRO}(d)$. $\mathcal{R}$ is an audience according to Lemma 1. Let us show that S is conflict-free w.r.t. $\mathcal{R}$. Since d is won by PRO, the length of d is equal to or greater than $|S|$ and so, according to Lemma 2, $S = \text{argPRO}(d) \setminus \{\_\}$. Moreover, since d is won by PRO, we have:

$$\phi_1(d) = \emptyset = \bigcup_{y \in \mathcal{A}^-_{\mathcal{R}}(\text{argPRO}(d)) \cap \text{argPRO}(d)} \{\langle y, \emptyset \rangle\}$$

so $\mathcal{A}^-_{\mathcal{R}}(\text{argPRO}(d)) \cap \text{argPRO}(d) = \emptyset$. In other words, no argument of argPRO(d)

is successfully attacked by argPRO(d) w.r.t. $\mathcal{R}$. Therefore argPRO(d), and then S, is conflict-free w.r.t. audience valPRO(d). $\square$

Property 2 (Completeness of ϕ_1) *Let $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* VAF. *If $S \subseteq \mathcal{X}$ is conflict-free w.r.t. at least one audience, and $S \neq \emptyset$, then there exists a ϕ_1-dialogue about S won by* PRO.

Notation 1 *Let $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* VAF. *Let $S \subseteq \mathcal{X}$. $\mathbb{V}(S)$ denotes the set of value preferences which makes 'unsuccessful' any successful (but not definite) attack between arguments of S, that is: $\mathbb{V}(S) = \bigcup_{x \in S, y \in S \text{ s.t. } \langle y,x \rangle \in \mathcal{A}} \{\langle \eta(x), \eta(y) \rangle\}$.*

Lemma 3 *Let $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* VAF. *A set $S \subseteq \mathcal{X}$ is conflict-free w.r.t. an audience $\mathcal{R}$ if and only if $\mathbb{V}(S) \subseteq \mathcal{R}^*$.*

Proof: Let $\mathcal{R}$ be an audience such that $\mathbb{V}(S) \subseteq \mathcal{R}^*$. Let $x \in S$ and $y \in S$ such that $\langle y, x \rangle \in \mathcal{A}$. y does not successfully attack x w.r.t. $\mathcal{R}$ since $\langle \eta(x), \eta(y) \rangle \in \mathbb{V}(S)$. So S is conflict-free w.r.t. $\mathcal{R}$. Now, let S be a conflict-free set w.r.t. an audience $\mathcal{R}$. For any $x \in S$ and $y \in S$, if $\langle y, x \rangle \in \mathcal{A}$, then $\langle \eta(x), \eta(y) \rangle \in \mathcal{R}^*$. So $\mathbb{V}(S) \subseteq \mathcal{R}^*$. $\square$

Lemma 4 *Let $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* VAF. *Let d be a ϕ_1-dialogue about a set $S \subseteq \mathcal{X}$, of the form $d = d'.$*[OPP, $\langle y, \emptyset \rangle$]*, and let $\mathcal{R} =$* valPRO(d)*. Then $\bigcup_{x \in \mathcal{A}^+_{\mathcal{R}}(y) \cap \text{argPRO}(d)} \{\langle \eta(x), \eta(y) \rangle\} \subseteq \mathbb{V}(S)$.*

Proof: The length of d is greater than $|S| + 1$ since the last move of d is by OPP. So, according to Lemma 2, argPRO(d) $= S \cup \{\_\}$. By definition, y belong to argPRO(d) and is not the empty argument, so $y \in S$. Any argument $x \in$ argPRO(d) successfully attacked by y w.r.t. $\mathcal{R}$, is not the empty argument, and belongs to S. So $\bigcup_{x \in \mathcal{A}^+_{\mathcal{R}}(y) \cap \text{argPRO}(d)} \{\langle \eta(x), \eta(y) \rangle\} \subseteq \bigcup_{x \in S, y \in S \text{ s.t. } \langle y,x \rangle \in \mathcal{A}} \{\langle \eta(x), \eta(y) \rangle\}$. $\square$

Lemma 5 *Let $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* VAF *and $S \subseteq \mathcal{X}$. If d is a ϕ_1-dialogue about S, then* valPRO(d) $\subseteq \mathbb{V}(S)$.

Proof: Let $S = \{a_1, \ldots, a_n\} \subseteq \mathcal{X}$. Let d be a ϕ_1-dialogue about S. If the length of d is lower than or equal to n, then all the moves of d have the form [PRO, $\langle a_i, \emptyset \rangle$] where a_i is an argument of S ($1 \leq i \leq n$). So valPRO(d) $= \emptyset$, and then valPRO(d) $\subseteq \mathbb{V}(S)$. Assume that if d is of a length k, with $k > n$, then valPRO(d) $\subseteq \mathbb{V}(S)$. Let us show that the property is true if d is of length $k+1$. d can have two forms. (1) If $d = d'.$[OPP, $\langle y, \emptyset \rangle$], then valPRO($d$) = valPRO($d'$). Since d' is of length k, valPRO(d') $\subseteq \mathbb{V}(S)$, and then valPRO(d) $\subseteq \mathbb{V}(S)$. (2) If $d = d'.$[PRO, $\langle \_, V \rangle$], then d' is of length k, and so valPRO(d') $\subseteq \mathbb{V}(S)$. According to Lemma 4, $V \subseteq \mathbb{V}(S)$. Consequently, since $\text{valPRO}(d') \cup V = \text{valPRO}(d)$, we have $\text{valPRO}(d) \subseteq \mathbb{V}(S)$. ▫

Lemma 6 *Let $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* VAF*. Let $S \subseteq \mathcal{X}$ be a conflict-free set for at least one audience, such that $S \neq \emptyset$. Let d be a ϕ_1-dialogue about S of length equal to or greater than $|S|$, the last move of which is played by* PRO*. If $\mathbb{V}(S) \not\subseteq (\text{valPRO}(d))^*$, then there exist $y \in \mathcal{X}$ and $V \subseteq \mathcal{V} \times \mathcal{V}$ such that $d.[\text{OPP}, \langle y, \emptyset \rangle].[\text{PRO}, \langle \_ , V \rangle]$ is a ϕ_1-dialogue.*

Proof: Let d be a ϕ_1-dialogue about S of length equal to or greater than $|S|$, the last move of which is played by PRO. Let $\mathcal{R} = \text{valPRO}(d)$. According to Lemma 1, $\mathcal{R}$ is an audience. According to Lemma 3, since $\mathbb{V}(S) \not\subseteq \mathcal{R}^*$, S is not conflict-free w.r.t. $\mathcal{R}$. Consequently, there exists $y \in \text{argPRO}(d)$ that successfully attacks some $x \in \text{argPRO}(d)$ w.r.t $\mathcal{R}$. In other words, there is some $\langle y, \emptyset \rangle \in \phi_1(d)$, and $d' = d.[\text{OPP}, \langle y, \emptyset \rangle]$ is a ϕ_1-dialogue. Now, let

$$V = \bigcup_{x \in \mathcal{A}^+_{\mathcal{R}}(y) \cap \text{argPRO}(d')} \{\langle \eta(x), \eta(y) \rangle\}.$$

Let us show that $\text{valPRO}(d') \cup V$ is an audience. According to Lemma 5, $\text{valPRO}(d') \subseteq \mathbb{V}(S)$. According to Lemma 4, $V \subseteq \mathbb{V}(S)$. So $\text{valPRO}(d) \cup V \subseteq \mathbb{V}(S)$. Let $\mathcal{T}$ be an audience w.r.t. which S is conflict-free. According to Lemma 3, $\mathbb{V}(S) \subseteq \mathcal{T}^*$. So $\text{valPRO}(d') \cup V \subseteq \mathcal{T}^*$ and then, $(\text{valPRO}(d') \cup V)^* \subseteq \mathcal{T}^*$. Consequently, $\text{valPRO}(d') \cup V$ is an audience. Hence, $\langle \_ , V \rangle \in \phi_1(d')$, and then $d.[\text{OPP}, \langle y, \emptyset \rangle].[\text{PRO}, \langle \_ , V \rangle]$ is a ϕ_1-dialogue. ▫

Proof: *(of Property 2)* Assume that S is a conflict-free set for at least one audience, and that $S \neq \emptyset$. Let $d_1 = [\text{PRO}, \langle a_1, \emptyset \rangle] \dots [\text{PRO}, \langle a_{|S|}, \emptyset \rangle]$ be the sequence of the first $|S|$ moves of a dialogue about S. Let, for $i \geq 2$,

$$d_i = d_{i-1}.[\text{OPP}, \langle y, \emptyset \rangle].[\text{PRO}, \langle \_ , V \rangle]$$

where, given that $\mathcal{R}_{i-1} = \text{valPRO}(d_{i-1}) = \text{valPRO}(d_{i-1}.[\text{OPP}, \langle y, \emptyset \rangle])$:

- $y \in \mathcal{A}^-_{\mathcal{R}_{i-1}}(\text{argPRO}(d_{i-1})) \cap \text{argPRO}(d_{i-1})$,
- $V = \bigcup_{x \in \mathcal{A}^+_{\mathcal{R}_{i-1}}(y) \cap \text{argPRO}(d_{i-1}.[\text{OPP}, \langle y, \emptyset \rangle])} \{\langle \eta(x), \eta(y) \rangle\}$, and
- $\text{valPRO}(d_i)$ is an audience.

Lemma 6 proves that the sequence is well-defined. Let us show that function ϕ_1 is strictly decreasing, that is, $\phi_1(d_i) \subset \phi_1(d_{i-1})$. Let $\mathcal{R}_i = \text{valPRO}(d_i)$. First, let us show that $\phi_1(d_i) \subseteq \phi_1(d_{i-1})$. Let

- $Z_i = \{z \in \mathcal{X} \mid \langle z, \emptyset \rangle \in \phi_1(d_i)\} = \mathcal{A}^-_{\mathcal{R}_i}(\text{argPRO}(d_i)) \cap \text{argPRO}(d_i)$,
- $Z_{i-1} = \{z \in \mathcal{X} \mid \langle z, \emptyset \rangle \in \phi_1(d_{i-1})\} = \mathcal{A}^-_{\mathcal{R}_{i-1}}(\text{argPRO}(d_{i-1})) \cap \text{argPRO}(d_{i-1})$.

According to Lemma 2, we have $Z_i = \mathcal{A}^-_{\mathcal{R}_i}(S) \cap S$ and $Z_{i-1} = \mathcal{A}^-_{\mathcal{R}_{i-1}}(S) \cap S$. Since $\text{valPRO}(d_{i-1}) \subseteq \text{valPRO}(d_i)$, $\mathcal{A}^-_{\mathcal{R}_i}(S) \subseteq \mathcal{A}^-_{\mathcal{R}_{i-1}}(S)$. So $Z_i \subseteq Z_{i-1}$ and hence $\phi_1(d_i) \subseteq \phi_1(d_{i-1})$.

Second, let us show that there exists some pair in $\phi_1(d_{i-1})$ that does not belong to $\phi_1(d_i)$. Consider the pair $\langle y, \emptyset\rangle$ which is in $\phi_1(d_{i-1})$. We have $y \in \mathcal{A}^-_{\mathcal{R}_{i-1}}(\text{argPRO}(d_{i-1})) \cap \text{argPRO}(d_{i-1})$. According to Lemma 2, $y \in \mathcal{A}^-_{\mathcal{R}_{i-1}}(S) \cap S$. Hence there exists some $x \in S$ such that $\langle y, x\rangle \in \mathcal{A}$ and $\langle \eta(x), \eta(y)\rangle \notin {\mathcal{R}_{i-1}}^*$. Since $\mathcal{R}_i = \mathcal{R}_{i-1} \cup V$, for any $x \in S$ such that $\langle y, x\rangle \in \mathcal{A}$, $\langle \eta(x), \eta(y)\rangle \in {\mathcal{R}_i}^*$. Consequently, $y \notin \mathcal{A}^-_{\mathcal{R}_i}(S) \cap S$, and hence $\langle y, \emptyset\rangle \notin \phi_1(d_i)$. So $\phi_1(d_i) \subset \phi_1(d_{i-1})$.
The empty set is the minimum of function ϕ_1, and for this set, the dialogue is won by PRO. □

This instance of the dialogue framework can indeed be used to check if the set of desired arguments of a DOR-partitioned VAF is conflict-free for at least one audience, and if so, to give such an audience. It is a corollary of the two previous properties.

Corollary 1 *Let $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta\rangle$ be a* DOR-VAF*. If d is a ϕ_1-dialogue about* $\text{Des}(\mathcal{X})$ *won by* PRO*, then* $\text{Des}(\mathcal{X})$ *is conflict-free w.r.t. audience* $\text{valPRO}(d)$*. If* $\text{Des}(\mathcal{X}) \neq \emptyset$ *is conflict-free w.r.t. at least one audience, then there exists a ϕ_1-dialogue about* $\text{Des}(\mathcal{X})$ *won by* PRO.

6.2.4 Making the arguments acceptable

Given a DOR-VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta\rangle$, let us assume that the set $\text{Des}(\mathcal{X})$ is conflict-free in the most restricted sense, that is, there are no arguments x and y in $\text{Des}(\mathcal{X})$ such that x attacks y. For $\mathcal{R}$ be an audience we call the set containing the desired arguments which aims at being a position the 'position under development'. The reason why the position under development would not be admissible w.r.t. $\mathcal{R}$ is that some arguments in it would not be acceptable to it w.r.t. $\mathcal{R}$, i.e. there is (at least one) argument x in the position under development such that some argument y successfully attacks x w.r.t. $\mathcal{R}$ and no argument z in the position successfully attacks y w.r.t. $\mathcal{R}$.

Let us consider a dialogue d about the conflict-free set $\text{Des}(\mathcal{X})$, based on a legal-move function where OPP plays moves where the argument is of the kind of y and the value ordering is empty, and where PRO plays in a way to make an argument such as x acceptable. The arguments of the position under development are those played by PRO. The value orderings played by PRO (i.e. $\text{valPRO}(d)$) must be the audience w.r.t. which the moves are made.

We identify four ways to make an argument x acceptable to the position under development:

(W1) Add to the position under development an optional argument z which definitely attacks y and which is not in conflict with any argument of the position under development.

(W2) Make the value of x preferred to the value of y, if x is not definitely attacked by y.

(W3) Add to the position under development an optional argument z which successfully but not definitely attacks y and which is not in conflict with any argument of the position under development.

(W4) Add to the position under development an optional argument z which successfully attacks y, and which might be successfully but not definitely attacked by the position under development or which might successfully but not definitely attack the position under development; the addition of value preferences to the current audience in order for the addition of z to the position to be correct must form an audience.

The following definition gives the formal translations of (W1) through (W4) as dialogue moves.

Definition 20 *Let* $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta \rangle$ *be a* DOR-VAF, d *be a dialogue about* $\mathrm{Des}(\mathcal{X})$, $\mathcal{R} = \mathrm{valPRO}(d)$. $\phi_2 : \mathcal{M}^* \to 2^{\mathcal{X}^- \times 2^{\mathcal{V} \times \mathcal{V}}}$ *is defined by:*

- *if* $\mathrm{pl}(\mathrm{last}(d)) = \mathrm{PRO}$ *(next move is by* OPP*), then*

$$\phi_2(d) = \bigcup_{y \in (\mathcal{A}^-_{\mathcal{R}}(\mathrm{argPRO}(d)) \setminus \mathcal{A}^+_{\mathcal{R}}(\mathrm{argPRO}(d)))} \{\langle y, \emptyset \rangle\};$$

- *if* $\mathrm{pl}(\mathrm{last}(d)) = \mathrm{OPP}$ *and* $\mathrm{arg}(\mathrm{last}(d)) = y$ *(next move is by* PRO*), let:*

$$Z_1 = (\mathrm{Opt}(\mathcal{X}) \cap \mathcal{A}^{--}_{\mathcal{R}}(y)) \setminus \mathcal{A}^{\pm}_{\mathcal{R}}(\mathrm{argPRO}(d)),$$
$$Z_2 = \mathrm{argPRO}(d) \cap (\mathcal{A}^{+}_{\mathcal{R}}(y) \setminus \mathcal{A}^{++}_{\mathcal{R}}(y)),$$
$$Z_3 = (\mathrm{Opt}(\mathcal{X}) \cap (\mathcal{A}^{-}_{\mathcal{R}}(y) \setminus \mathcal{A}^{--}_{\mathcal{R}}(y))) \setminus \mathcal{A}^{\pm}_{\mathcal{R}}(\mathrm{argPRO}(d)),$$
$$Z_4 = \{z \in Z'_4 \mid \mathcal{R} \cup V_{XY(z)} \text{ is an audience}\} \text{ with:}$$
$$Z'_4 = (\mathrm{Opt}(\mathcal{X}) \cap \mathcal{A}^{-}_{\mathcal{R}}(y)) \cap ((\mathcal{A}^{+}_{\mathcal{R}}(\mathrm{argPRO}(d)) \setminus \mathcal{A}^{++}_{\mathcal{R}}(\mathrm{argPRO}(d))) \cup (\mathcal{A}^{-}_{\mathcal{R}}(\mathrm{argPRO}(d)) \setminus \mathcal{A}^{--}_{\mathcal{R}}(\mathrm{argPRO}(d)))),$$

and, given $z \in Z'_4$*:*

$$X(z) = \mathrm{argPRO}(d) \cap (\mathcal{A}^{-}_{\mathcal{R}}(z) \setminus \mathcal{A}^{--}_{\mathcal{R}}(z)),$$
$$Y(z) = \mathrm{argPRO}(d) \cap (\mathcal{A}^{+}_{\mathcal{R}}(z) \setminus \mathcal{A}^{++}_{\mathcal{R}}(z)),$$
$$V_{XY(z)} = \begin{cases} (\bigcup_{x \in X(z)} \{\langle \eta(z), \eta(x) \rangle\} \cup \bigcup_{w \in Y(z)} \{\langle \eta(w), \eta(z) \rangle\} \\ \quad \cup \{\langle \eta(z), \eta(y) \rangle\}) & \text{if } \eta(z) \neq \eta(y) \text{ and } \langle \eta(z), \eta(y) \rangle \notin (\mathcal{R})^*, \\ (\bigcup_{x \in X(z)} \{\langle \eta(z), \eta(x) \rangle\} \cup \bigcup_{w \in Y(z)} \{\langle \eta(w), \eta(z) \rangle\}) & \text{otherwise.} \end{cases}$$

Then:

(W1) *if* $Z_1 \neq \emptyset$, *then* $\phi_2(d) = \bigcup_{z \in Z_1} \{\langle z, \emptyset \rangle\}$,

or **(W2)** *if* $Z_2 \neq \emptyset$, *then* $\phi_2(d) = \bigcup_{z \in Z_2} \{\langle \_ , \{\langle \eta(z), \eta(y) \rangle\} \rangle\}$,

or **(W3)** *if* $Z_3 \neq \emptyset$, *then* $\phi_2(d) = \bigcup_{z \in Z_3} \{\langle z, \{\langle \eta(z), \eta(y) \rangle\} \rangle\}$,

or **(W4)** *if* $Z_4 \neq \emptyset$*, then* $\phi_2(d) = \bigcup_{z \in Z_4}\{\langle z, V_{XY(z)}\rangle\}$*,*
or *if* $Z_1 \cup Z_2 \cup Z_3 \cup Z_4 = \emptyset$*, then* $\phi_2(d) = \emptyset$*.*

Each of these four ways would be tried in turn. In responding to an attack, the proponent will wish to maintain as much flexibility to respond to further attacks as possible. The order in which the four ways are tried is thus determined by the desire to make the least committal move at any stage. Flexibility is limited in two ways. If the position is extended by including an additional argument, as in W1, W3 and W4, the potential attackers of the position is increased since this argument must now also be defended by the position. If a commitment to a value ordering is made, as in W2, W3 and W4, this must be subsequently respected, which restricts the scope to make such moves in future responses. We regard this second line of defence as more committal that the first. Therefore W1 should be tried first since it imposes no constraints on the audience, although it does extend the position. W2 should be selected next because, although it does constrain the audience to adopt a certain value preference, it does not introduce any additional arguments to the position, and so does not give rise to any additional attackers. If W3 is resorted to, both the position is extended and a value ordering commitment is made, but the argument introduced is compatible with the existing position. W4 should be the final resort because it extends the position, constrains the audience, and requires further constraints to be imposed to enable it to cohere with the existing position.

The dialogue framework instantiated with the legal-move function ϕ_2, is correct and complete w.r.t. the determination of an audience for which the conflict-free set of desired arguments is admissible for at least one audience:

Property 3 (Soundness of ϕ_2) *Let* $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta\rangle$ *be a* DOR-VAF *such that* $\mathrm{Des}(\mathcal{X})$ *is conflict-free. If* d *is a* ϕ_2*-dialogue about* $\mathrm{Des}(\mathcal{X})$ *won by* PRO*, then* $\mathrm{argPRO}(d)\backslash\{\_\}$ *is a position such that* $\mathrm{valPRO}(d)$ *is a corresponding audience.*

Lemma 7 *Let* $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta\rangle$ *be a* VAF*. Let* $v_1, v_2 \in \mathcal{V}$ *and* $\mathcal{R}$ *an audience. If* $\langle v_1, v_2\rangle \notin \mathcal{R}^*$ *and* $v_1 \neq v_2$*, then* $\mathcal{R} \cup \{\langle v_2, v_1\rangle\}$ *is an audience.*

Proof: Let us assume that $\langle v_1, v_2\rangle \notin \mathcal{R}^*$ and $v_1 \neq v_2$. If $\langle v_2, v_1\rangle \in \mathcal{R}$, then obviously, $\mathcal{R} \cup \{\langle v_2, v_1\rangle\}$ is an audience. If $\langle v_2, v_1\rangle \notin \mathcal{R}$, let us assume that $\mathcal{R} \cup \{\langle v_2, v_1\rangle\}$ is not an audience. Therefore, there would exist $v \in \mathcal{V}$ such that $\langle v, v\rangle \in (\mathcal{R} \cup \{\langle v_2, v_1\rangle\})^*$. Since $v_1 \neq v_2$, we know that $\langle v_2, v_1\rangle \neq \langle v, v\rangle$. So $\langle v, v\rangle \notin \mathcal{R} \cup \{\langle v_2, v_1\rangle\}$, but $\langle v, v\rangle \in (\mathcal{R} \cup \{\langle v_2, v_1\rangle\})^*$. Hence, there would be in $\mathcal{R}$ a set of pairs such that:

$$\{\langle v, x_1\rangle, \langle x_1, x_2\rangle, \ldots, \langle x_{i-1}, x_i\rangle, \langle x_i, v_2\rangle, \langle v_1, x_{i+1}\rangle, \langle x_{i+1}, x_{i+2}\rangle, \ldots, \langle x_n, v\rangle\}$$

for some $n \geq 0$. Then $\mathcal{R}^*$ would contain $\langle v, v_2\rangle$, $\langle v_1, v\rangle$, and $\langle v_1, v_2\rangle$. This is not possible since we have made the assumption that $\langle v_1, v_2\rangle \notin \mathcal{R}^*$. Consequently, such a sequence does not exist and $\mathcal{R} \cup \{\langle v_2, v_1 \rangle\}$ is an audience. $\square$

Lemma 8 *Given a* DOR-VAF $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta \rangle$ *such that* $\text{Des}(\mathcal{X}) \neq \emptyset$ *is conflict-free, and a finite* ϕ_2*-dialogue* d*,* $\text{argPRO}(d)$ *is conflict-free w.r.t. the audience* $\text{valPRO}(d)$*.*

Proof: We prove the result by induction on the number of elements of $\text{argPRO}(d)$. If $\text{argPRO}(d)$ only contains $\text{Des}(\mathcal{X})$, $\text{valPRO}(d) = \emptyset$, then $\text{argPRO}(d)$ is conflict-free w.r.t. the audience $\text{valPRO}(d)$. Suppose now that the property is true for any ϕ_2-dialogue d such that $\text{argPRO}(d)$ contains at most $n-1$ elements, for some $n > |\text{Des}(\mathcal{X})|$. Let d be a ϕ_2-dialogue such that $\text{argPRO}(d)$ contains n elements. Suppose first that the last move of d is played by PRO: d has the form $d = d'.[\text{OPP}, \langle y, \emptyset \rangle].[\text{PRO}, \langle z, V \rangle]$, where $y \in \mathcal{A}^-_{\text{valPRO}(d')}(\text{argPRO}(d')) \backslash \mathcal{A}^+_{\text{valPRO}(d')}(\text{argPRO}(d'))$ and, given $\mathcal{R} = \text{valPRO}(d') = \text{valPRO}(d'.[\text{OPP}, \langle y, \emptyset \rangle])$ and $d'' = d'.[\text{OPP}, \langle y, \emptyset \rangle]$, either:

- $\langle z, V \rangle$ is played according to W1; in this case, $z \in (\text{Opt}(\mathcal{X}) \cap \mathcal{A}^{--}_{\mathcal{R}}(y)) \setminus \mathcal{A}^{\pm}_{\mathcal{R}}(\text{argPRO}(d''))$ and $V = \emptyset$. Since $z \notin \mathcal{A}^{\pm}_{\mathcal{R}}(\text{argPRO}(d''))$, z is conflict-free w.r.t. $\mathcal{R}$ with $\text{argPRO}(d'')$, and $\mathcal{R} \cup V = \mathcal{R}$ is an audience. Moreover, since $y \notin \mathcal{A}^+_{\mathcal{R}}(\text{argPRO}(d'))$ and $y \in \mathcal{A}^{++}_{\mathcal{R}}(z)$, $z \notin \text{argPRO}(d')$. Consequently, d' contains strictly less than n elements. Thus, by induction hypothesis, $\text{argPRO}(d')$ is conflict-free w.r.t. $\mathcal{R}$. Hence, $\text{argPRO}(d') \cup \{z\}$ is conflict-free w.r.t. $\mathcal{R} \cup V$.
- $\langle z, V \rangle$ is played according to W2; in this case, $z = \_$ and $V = \langle \eta(t), \eta(y) \rangle$ for some $t \in \text{argPRO}(d'') \cap (\mathcal{A}^+_{\mathcal{R}}(y) \setminus \mathcal{A}^{++}_{\mathcal{R}}(y))$. z is obviously conflict-free with $\text{argPRO}(d'')$ w.r.t. $\mathcal{R}$ and $\mathcal{R} \cup V$. Moreover, since $t \in \mathcal{A}^+_{\mathcal{R}}(y) \setminus \mathcal{A}^{++}_{\mathcal{R}}(y)$, we have $\langle \eta(y), \eta(t) \rangle \notin (\mathcal{R})^*$ and $\eta(t) \neq \eta(y)$. So, by Lemma 7, $\mathcal{R} \cup V$ is an audience. Assuming that the argument z is different from any argument played in $\text{argPRO}(d')$ (and especially any $\_$), then d' contains strictly less than n elements. Thus, by induction hypothesis, $\text{argPRO}(d')$ is conflict-free w.r.t. $\mathcal{R}$. Hence, $\text{argPRO}(d') \cup \{z\}$ is conflict-free w.r.t. $\mathcal{R} \cup V$.
- $\langle z, V \rangle$ is played according to W3; in this case, $z \in (\text{Opt}(\mathcal{X}) \cap (\mathcal{A}^-_{\mathcal{R}}(y) \setminus \mathcal{A}^{--}_{\mathcal{R}}(y))) \backslash \mathcal{A}^{\pm}_{\mathcal{R}}(\text{argPRO}(d''))$, and $V = \langle \eta(z), \eta(y) \rangle$. Since $z \notin \mathcal{A}^{\pm}_{\mathcal{R}}(\text{argPRO}(d''))$, z is conflict-free w.r.t. $\mathcal{R}$ with $\text{argPRO}(d'')$. Moreover, since $z \in \mathcal{A}^-_{\mathcal{R}}(y) \setminus \mathcal{A}^{--}_{\mathcal{R}}(y)$, $\langle \eta(y), \eta(z) \rangle \notin (\mathcal{R})^*$ and $\eta(y) \neq \eta(z)$. So, by Lemma 7, $\mathcal{R} \cup V$ is an audience. Now, since $y \notin \mathcal{A}^+_{\mathcal{R}}(\text{argPRO}(d'))$ and $y \in \mathcal{A}^+_{\mathcal{R}}(z)$, $z \notin \text{argPRO}(d')$. Consequently, d' contains strictly less than n elements. Thus, by induction hypothesis, $\text{argPRO}(d')$ is conflict-free w.r.t. $\mathcal{R}$. Hence, $\text{argPRO}(d') \cup \{z\}$ is conflict-free w.r.t. $\mathcal{R} \cup V$.
- $\langle z, V \rangle$ is played according to W4; in this case, $z \in (\text{Opt}(\mathcal{X}) \cap \mathcal{A}^-_{\mathcal{R}}(y)) \cap ((\mathcal{A}^+_{\mathcal{R}}(\text{argPRO}(d'')) \backslash \mathcal{A}^{++}_{\mathcal{R}}(\text{argPRO}(d''))) \cup (\mathcal{A}^-_{\mathcal{R}}(\text{argPRO}(d'')) \backslash \mathcal{A}^{--}_{\mathcal{R}}(\text{argPRO}(d''))))$ and $V = V_{XY(z)}$ such that $\mathcal{R} \cup V_{XY(z)}$ is an audience. z is in conflict with $\text{argPRO}(d'')$ w.r.t. $\mathcal{R}$, but not w.r.t. $\mathcal{R} \cup V$: actually, for any argument $x \in \text{argPRO}(d'')$ that successfully but not definitely attacks z w.r.t. $\mathcal{R}$ (i.e.

$x \in X(z)$), $\langle \eta(z), \eta(x) \rangle \in V$, and for any argument $w \in$ argPRO(d'') that is successfully but not definitely attacked by z w.r.t. $\mathcal{R}$ (i.e. $w \in Y(z)$), then $\langle \eta(w), \eta(z) \rangle \in V$. Now, since $y \notin \mathcal{A}^+_{\mathcal{R}}(\text{argPRO}(d'))$ and $y \in \mathcal{A}^+_{\mathcal{R}}(z)$, $z \notin$ argPRO(d'). Consequently, d' contains strictly less than n elements. Thus, by induction hypothesis, argPRO(d') is conflict-free w.r.t. $\mathcal{R}$. Hence, argPRO$(d') \cup \{z\}$ is conflict-free w.r.t. $\mathcal{R} \cup V$.

Since argPRO$(d') \cup \{z\} =$ argPRO(d), and $\mathcal{R} \cup V =$ valPRO(d), argPRO(d) is conflict-free w.r.t. the audience valPRO(d). Suppose now that the last move is played by OPP. Then d has the form $d = d'.[\text{OPP}, \langle y, \emptyset \rangle]$ where d' is a ϕ_2-dialogue such that argPRO$(d') =$ argPRO(d) and valPRO$(d') =$ valPRO(d). Therefore, argPRO(d') contains n elements. We have just proved that in this case, argPRO(d') is conflict-free w.r.t. valPRO(d'), hence argPRO(d) is w.r.t. valPRO(d). □

Proof: *(of Property 3)* Let d be a ϕ_2-dialogue about Des$(\mathcal{X})$ won by PRO. According to Lemma 8, argPRO(d) is conflict-free w.r.t. audience valPRO(d). Since d is won by PRO, $\phi_2(d) = \emptyset$, and hence:

$$\mathcal{A}^-_{\text{valPRO}(d)}(\text{argPRO}(d)) \setminus \mathcal{A}^+_{\text{valPRO}(d)}(\text{argPRO}(d)) = \emptyset.$$

In other words, every argument in argPRO(d) is acceptable to argPRO(d) w.r.t. valPRO(d). Consequently, argPRO(d) is admissible w.r.t. valPRO(d). Since d is a ϕ_2-dialogue about Des$(\mathcal{X})$, Des$(\mathcal{X}) \subseteq$ argPRO(d). Any optional argument played by PRO in d is used to make acceptable another argument of argPRO(d) that would not be otherwise acceptable to argPRO(d). No rejected argument is played by PRO. The empty argument has no role in the admissibility of argPRO(d). Consequently, argPRO$(d) \setminus \{_\}$ is a position, and valPRO(d) is a corresponding audience. □

Property 4 (Completeness of ϕ_2) *Let $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* DOR-VAF *such that* Des$(\mathcal{X}) \neq \emptyset$ *is conflict-free. If $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta \rangle$ has at least one position, then there exists a ϕ_2-dialogue about* Des$(\mathcal{X})$ *won by* PRO*.*

Lemma 9 *Let $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta \rangle$ be a* DOR-VAF *such that* Des$(\mathcal{X}) \neq \emptyset$ *is conflict-free. Let d be a ϕ_2-dialogue of length greater than* $|$Des$(\mathcal{X})|$*, the last move of which is played by* PRO*. Let S be a minimal position that contains* argPRO$(d) \setminus \{_\}$*, and $\mathcal{R}$ be a minimal corresponding audience of S. If $S \neq$* argPRO$(d) \setminus \{_\}$ *or $(\mathcal{R})^* \neq$* (valPRO$(d))^*$*, then there exist $y \in \mathcal{X}$, $z \in \mathcal{X}^-$ and $V \subseteq \mathcal{V} \times \mathcal{V}$ such that the dialogue $d' = d.[\text{OPP}, \langle y, \emptyset \rangle].[\text{PRO}, \langle z, V \rangle]$ is a ϕ_2-dialogue and S is minimal such that S is a position and contains* argPRO$(d') \setminus \{_\}$*, and $\mathcal{R}$ is a minimal corresponding audience of S that contains* valPRO(d')*.*

Proof: Since argPRO$(d) \setminus \{_\} \neq S$ or (valPRO$(d))^* \neq \mathcal{R}^*$, and since argPRO$(d) \setminus \{_\} \subseteq S$, (valPRO$(d))^* \subseteq \mathcal{R}^*$, the minimality of S and the corresponding audience $\mathcal{R}$ imply that argPRO(d) is not admissible w.r.t. valPRO(d). From Lemma 8, we know that argPRO(d) is conflict-free w.r.t. valPRO(d). Thus $\mathcal{A}^-_{\text{valPRO}(d)}(\text{argPRO}(d)) \setminus \mathcal{A}^+_{\text{valPRO}(d)}(\text{argPRO}(d)) \neq \emptyset$. Hence, $\phi_2(d) \neq \emptyset$. Let $y \in \mathcal{A}^-_{\text{valPRO}(d)}(\text{argPRO}(d)) \setminus \mathcal{A}^+_{\text{valPRO}(d)}(\text{argPRO}(d))$. Since $\text{argPRO}(d) \setminus \{\_\} \subseteq S$, $(\text{valPRO}(d))^* \subseteq \mathcal{R}^*$, and S admissible w.r.t. $\mathcal{R}$, $\exists z \in \{\_\} \cup (S \cap \text{Opt}(\mathcal{X}))$ and $\exists V \subseteq \mathcal{R}^*$ such that, either:

- $y \in \mathcal{A}^{++}_{\text{valPRO}(d)}(z)$; in this case, $z \in Z_1$, $V = \emptyset$.
- $V = \langle \eta(t), \eta(y) \rangle$ for $t \in \text{argPRO}(d) \cap (\mathcal{A}^+_{\text{valPRO}(d)}(y) \setminus \mathcal{A}^{++}_{\text{valPRO}(d)}(y)) = Z_2$ and $z = \_$.
- $y \in \mathcal{A}^+_{\text{valPRO}(d)}(z) \setminus \mathcal{A}^{++}_{\text{valPRO}(d)}(z)$ and $V = \langle \eta(z), \eta(y) \rangle$; in this case, $z \in Z_3$.
- $y \in \mathcal{A}^+_{\text{valPRO}(d)}(z)$ and z is in conflict with argPRO(d) w.r.t. valPRO(d), but, for any argument x that successfully but not definitely attacks z, $\langle \eta(z), \eta(x) \rangle \in V$, for any argument w that is successfully but not definitely attacked by z, $\langle \eta(w), \eta(z) \rangle \in V$, and if $\eta(z) \neq \eta(y)$, $\langle \eta(z), \eta(y) \rangle \in V$; in this case, $z \in Z_4$.

Consequently, $\langle z, V \rangle \in \phi_2(d.[\text{OPP}, \langle y, \emptyset \rangle])$. Thus, $d' = d.[\text{OPP}, \langle y, \emptyset \rangle].[\text{PRO}, \langle z, V \rangle]$ is a ϕ_2-dialogue.
We know that $\text{argPRO}(d') = \text{argPRO}(d) \cup \{z\}$, $\text{argPRO}(d') \setminus \{\_\} \subseteq S$, and $(\text{valPRO}(d'))^* = (\text{valPRO}(d) \cup V)^* \subseteq \mathcal{R}^*$. There remains to prove that no set $S' \subset S$ that contains $\text{argPRO}(d') \setminus \{\_\}$, and no $\mathcal{R}' \subseteq \mathcal{V} \times \mathcal{V}$ such that $(\text{valPRO}(d'))^* \subseteq (\mathcal{R}')^* \subset \mathcal{R}^*$, are such that:

• S' is admissible w.r.t. $\mathcal{R}$. Suppose that such a set S' exists. Then $\text{argPRO}(d) \setminus \{\_\} \subseteq \text{argPRO}(d') \setminus \{\_\}$. Hence, $\text{argPRO}(d) \setminus \{\_\} \subset S$. Since S is minimal such that S contains $\text{argPRO}(d) \setminus \{\_\}$ and S is admissible w.r.t. $\mathcal{R}$, S' is not admissible w.r.t. $\mathcal{R}$.
• $\mathcal{R}'$ is an audience w.r.t. which S is admissible. Suppose that such an $\mathcal{R}'$ exists. Then $(\text{valPRO}(d))^* \subseteq (\text{valPRO}(d'))^*$, and hence $(\text{valPRO}(d))^* \subset \mathcal{R}^*$. Since $\mathcal{R}$ is minimal such that $\mathcal{R}^*$ contains $(\text{valPRO}(d))^*$ and $\mathcal{R}$ is an audience w.r.t. which S is admissible, $\mathcal{R}'$ is not an audience. □

Proof: *(of Property 4)* Let S be a minimal subset of $\mathcal{X}$ such that S is a position, and $\mathcal{R}$ be a minimal corresponding audience of S. Given $\text{Des}(\mathcal{X}) = \{a_1, \ldots, a_n\}$, let $d_1 = [\text{PRO}, \langle a_1, \emptyset \rangle] \ldots [\text{PRO}, \langle a_n, \emptyset \rangle]$. Given $j > 1$, let $d_j = d_{j-1}.[\text{OPP}, \langle y, \emptyset \rangle].[\text{PRO}, \langle z, V \rangle]$ if $\text{argPRO}(d_{j-1}) \neq S$ and $(\text{valPRO}(d_{j-1}))^* \neq \mathcal{R}^*$; $\langle y, \emptyset \rangle \in \phi_2(d_{j-1})$ and $\langle z, V \rangle \in \phi_2(d_{j-1}.[\text{OPP}, \langle y, \emptyset \rangle])$. Lemma 9 proves that the sequence is well defined, and that, when $\text{argPRO}(d_j) = S$ and $(\text{valPRO}(d_j))^* = \mathcal{R}^*$, there exists $j \geq 1$ such that d_j is a ϕ_2-dialogue about $\text{Des}(\mathcal{X})$ won by PRO (since S is admissible w.r.t. $\mathcal{R}$, $\mathcal{A}^-_{\text{valPRO}(d_j)} \setminus \mathcal{A}^+_{\text{valPRO}(d_j)} = \emptyset$). □

6.2.5 Development of positions

Let us consider the following legal-move function:

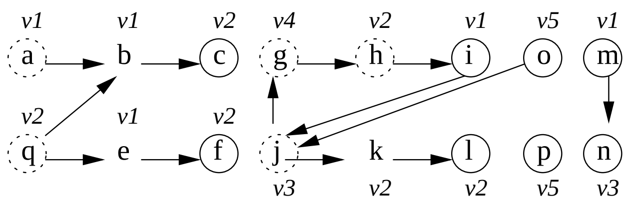
Definition 21 *Let* $\langle \mathcal{X}^-, \mathcal{A}, \mathcal{V}, \eta \rangle$ *be a* DOR-VAF*,* d *be a dialogue about* $\mathrm{Des}(\mathcal{X})$*.* $\phi_3 : \mathcal{M}^* \to 2^{\mathcal{X}^- \times 2^{\mathcal{V} \times \mathcal{V}}}$ *is defined by:*

- *if* $\mathrm{pl}(\mathrm{last}(d)) = \mathrm{PRO}$ *(next move is by* OPP*), then, if* $\phi_1(d) \neq \emptyset$*, then* $\phi_3(d) = \phi_1(d)$ *else* $\phi_3(d) = \phi_2(d)$*;*
- *if* $\mathrm{pl}(\mathrm{last}(d)) = \mathrm{OPP}$ *(next move is by* PRO*), if* $\mathrm{arg}(\mathrm{last}(d)) \in \mathrm{Des}(\mathcal{X})$ *then* $\phi_3(d) = \phi_1(d)$*, else* $\phi_3(d) = \phi_2(d)$*.*

Property 5 *Let* $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ *be a* DOR-VAF*. If* d *is a* ϕ_3*-dialogue about* $\mathrm{Des}(\mathcal{X})$ *won by* PRO*, then* $\mathrm{argPRO}(d) \setminus \{\_\}$ *is a position such that* $\mathrm{valPRO}(d)$ *is a corresponding audience. If* $\mathrm{Des}(\mathcal{X}) \neq \emptyset$ *is contained in a position, then there exists a* ϕ_3*-dialogue about* $\mathrm{Des}(\mathcal{X})$ *won by* PRO*.*

Proof: Consequence of Corollary 1, Property 3 and Property 4. ▫

Example Consider the following VAF $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$:



The arguments are the vertices of the graph and the edges represent the elements of the attack relation. The set of values is $\mathcal{V} = \{v1, v2, v3, v4, v5\}$. The value associated to an argument is indicated just below or just above the argument. The desired arguments are plain-circled, the optional arguments are dot-circled, and the rejected arguments are not circled. Let us develop a position. We start a ϕ_3-dialogue d about $\mathrm{Des}(\mathcal{X})$. The first moves of d contain the desired arguments, i.e. $\mu_{0_1}\mu_{0_2}\mu_{0_3}\mu_{0_4}\mu_{0_5}\mu_{0_6}\mu_{0_7}\mu_{0_8} = [\mathrm{PRO}, \langle c, \emptyset \rangle][\mathrm{PRO}, \langle f, \emptyset \rangle]$ $[\mathrm{PRO}, \langle i, \emptyset \rangle]$ $[\mathrm{PRO}, \langle l, \emptyset \rangle][\mathrm{PRO}, \langle m, \emptyset \rangle][\mathrm{PRO}, \langle n, \emptyset \rangle][\mathrm{PRO}, \langle o, \emptyset \rangle][\mathrm{PRO}, \langle p, \emptyset \rangle]$. Then, to ensure the conflict-freeness of $\mathrm{Des}(\mathcal{X})$ w.r.t. one audience:

$\mu_1 = [\mathrm{OPP}, \langle m, \emptyset \rangle]$
$\mu_2 = [\mathrm{PRO}, \langle \_ , \{\langle v3, v1 \rangle\} \rangle]$

Now, to make the arguments of $\mathrm{Des}(\mathcal{X})$ acceptable:

$\mu_3 = [\mathrm{OPP}, \langle b, \emptyset \rangle]$
$\mu_4 = [\mathrm{PRO}, \langle a, \emptyset \rangle]$ **(W1)**
$\mu_5 = [\mathrm{OPP}, \langle e, \emptyset \rangle]$
$\mu_6 = [\mathrm{PRO}, \langle \_ , \{\langle v2, v1 \rangle\} \rangle]$ **(W2)**
$\mu_7 = [\mathrm{OPP}, \langle h, \emptyset \rangle]$

$\mu_8 = [\text{PRO}, \langle g, \{\langle v4, v2\rangle\}\rangle]$ **(W3)**
$\mu_9 = [\text{OPP}, \langle j, \emptyset\rangle]$
$\mu_{10} = [\text{PRO}, \langle \_ , \{\langle v4, v3\rangle\}\rangle]$ **(W2)**
$\mu_{11} = [\text{OPP}, \langle k, \emptyset\rangle]$
$\mu_{12} = [\text{PRO}, \langle j, \{\langle v3, v2\rangle, \langle v3, v5\rangle\}\rangle]$ **(W4)**

$d = \mu_{0_1} \ldots \mu_{0_8}\mu_1\mu_2\mu_3\mu_4\mu_5\mu_6\mu_7\mu_8\mu_9\mu_{10}\mu_{11}\mu_{12}$ is a ϕ_3-dialogue won by PRO. The set $\text{argPRO}(d) = \text{Des}(\mathcal{X}) \cup \{a, g, j\}$ is a position, and $\text{valPRO}(d) = \{\langle v4, v3\rangle, \langle v3, v2\rangle, \langle v4, v2\rangle, \langle v2, v1\rangle, \langle v3, v1\rangle, \langle v3, v5\rangle\}$ is one of its corresponding audiences.

At certain points we may be presented with a choice of arguments to use with W1-4. For example b may be attacked by a or, if $v1$ is not preferred to $v2$, q. Similarly there are choices when we declare value preferences: in the example we can either prevent the attack of j on g succeeding, or choose preferences which lead to i or o defeating j. Such choices may, if badly made, lead to backtracking. Some heuristics seem possible to guide choices: it is better to attack an undesired argument with an argument of its own value where possible, as with a and b above, as this attack will succeed even if the value order changes. Also, when a value preference is required, a choice which keeps an optional argument available is better than one which defeats it, as the argument may be required to defeat a future attack, as in the example where j is required to defeat k.

7 Related work

7.1 Acceptance

In this subsection our main purpose is to clarify the relation between the concepts of subjective (objective) acceptance and credulous (sceptical) acceptance as used in standard argument systems. In addition we consider the behaviour of the algorithm FIND AUDIENCE in rather more discursive terms relating it to the problems of testing a set arguments for admissibility or stability as defined in Defn 1.

In earlier work, e.g. [16, p. 369], the Argument Systems of Dung [12] have been considered as VAFs in which a *single* value is associated with all arguments (e.g. "truth"). We argue that a rather more subtle interpretation – also relevant to comparisons with the *preference-based* schema introduced by Amgoud and Cayrol [1] – is appropriate.

We remarked earlier that Defn. 1 and Defn. 5 describe equivalent structures when the underlying audience is $\mathcal{R} = \emptyset$, i.e. the *universal audience*. Thus one could view the apparent shift from intractability in Dung's framework, e.g. as evidenced by the results of [10,14,15], to the polynomial-time procedures available for VAFs, e.g. as described in Fact 6, Theorem 11 as indicative of how increased awareness of the underlying relationships offering reasons for acceptance of arguments within an argument system can assist in resolving issues. In order to amplify this point, rather than treating the standard systems of [12] as "VAFs in which only a single value is present", we may consider these as VAFs (in the sense of Defn. 2) but in which one has (initially) *no knowledge* regarding the values associated with arguments or the relative orderings of these values that are held by protagonists. Thus, while one has (it may be presumed) agreement on the set of values ($\mathcal{V}$) germane to the framework and on the manner in which these relate to individual arguments – the mapping $\eta : \mathcal{X} \to \mathcal{V}$ – in the absence of any indication of value priorities, the case for some argument, x say, being acceptable can only be "rationalised" in terms of the assumption that "every attack is successful". In other words the effective audience is $\mathcal{R} = \emptyset$ – the universal audience – and an attack by y on x must, *ceteribus paribus*, be deemed to succeed, even when $\eta(y) \neq \eta(x)$, since a rational disputant has no basis to reject the attack $\langle y, x \rangle$ *in itself*: to promote x its defenders must subsequently resort to finding attacks on y. Within the VAF framework, however, defenders of x have a rationale for rejecting the attack by y when $\eta(y) \neq \eta(x)$: by indicating that they subscribe *only* to audiences wherein $\eta(x) \succ_{\mathcal{R}} \eta(y)$ so that the attack $\langle y, x \rangle$ is unsuccessful with respect to such audiences. In this way further debate concerning x takes place not in the context of the universal audience but in the context of the audience $\mathcal{R} = \{\langle \eta(x), \eta(y) \rangle\}$.

In summary, the revelation of successive value orderings, assuming these to be consistent in that $\mathcal{R}$ remains an audience, may lead eventually to $\mathcal{R}$ being a *specific* audience: from the initial analysis of $\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$, knowledge and conditions on $\mathcal{R}$ have evolved from the state where nothing is assumed – $\mathcal{R} = \emptyset$ – to one in which a specific audience has been revealed, yielding *exactly one* (rational with respect to this audience) interpretation of which arguments are acceptable. We observe that in moving from $\mathcal{R} = \emptyset$, the computational effect of adding $\langle v, v' \rangle$ to $\mathcal{R}$ is modelled by removing from the directed graph stucture $\langle \mathcal{X}, \mathcal{A} \rangle$ all edges $\langle x, y \rangle$ for which $\eta(x) = v'$ and $\eta(y) = v$, so that (given the presence of at least two different values in any directed cycle of $\langle \mathcal{X}, \mathcal{A} \rangle$) specific audiences will result in some directed *acyclic* graph structure: [7] that such forms have a unique, non-empty preferred extension is immediate from [12, Thm. 30], whose proof easily yields a polynomial-time algorithm for its construction.

[7] It is, of course, possible that the reduction of $\mathcal{A}$ induced by an audience $\mathcal{R}$ defines an acyclic graph without $\mathcal{R}$ being a specific audience.

We note that the preference-based approach of [1] can also be treated as a mechanism to explain progression from an unrestricted argument system to an acyclic form: the response to an attack by y on x being that the argument x "is preferred to" the argument y and thence the attack $\langle y, x\rangle$ may be eliminated from $\langle \mathcal{X}, \mathcal{A}\rangle$ so that following the declaration of some finite number of preferences the resulting graph is acyclic. Unlike the interaction between Defn. 1 and Defn. 5 which we may relate via VAFs and the universal audience, analogues between VAF and preference-based schemes are less clearly defined: while it is certainly the case that preference-based frameworks can be interpreted in terms of VAFs in which every argument is associated with a *unique* value [8] such an approach is unappealing, although it does emphasise that in such frameworks, unlike VAFs, the expression of a preference for one argument over another has no implications for other choices that need to be made: in scenarios where VAF structures have been used, e.g. the suite of legal examples presented in [4], typically the number of distinct values is "small" relative to the number of arguments. A further important distinction between these two models can be seen in terms of our earlier discussion of mechanisms for responding to an attack on x by y. Thus, in all three schemes, i.e. those of [1,3,12] – one option is to counterattack y; in [1,3] there is the further possibility of "not recognising" the attack $\langle y, x\rangle$. In [1] a preference for x over y is expressed: these, however, are not "explained" and do not have implications for subsequent preferences that might be indicated. [9] The VAF approach, however, requires an additional rationale for such a preference to be given: the attack by y on x fails because the defender of x regards its associated value, $\eta(x)$, as having greater importance than that of its attacker. One significant consequence of this implicit justification is that its speaker must act consistently regarding other attacks, e.g. an argument z with $\eta(z) = \eta(y)$ can not be used to counterattack an argument w with $\eta(w) = \eta(x)$.

7.2 Hunter's notion of impact for an audience

Another approach in which computational use of made of the notion of audience is that of Hunter [18]. Hunter adopts a notion of argument in which an argument is a pair comprising a set of formulae (the *support*) and a formula (the *consequent*) which can be classically derived from the support. Different

[8] In the same way, VAFs can be considered as preference-based frameworks where the argument preference relation is determined via the value orderings. Where the preference relation supplies a total order, each argument will need to have a distinct value.

[9] That is, other than the requirement that the irreflexive transitive closure of the preference relation be asymettric, e.g. given three arguments x, y, and z with $\mathcal{A} = \{\langle x, z\rangle, \langle z, y\rangle, \langle y, x\rangle\}$ it cannot be claimed (within the schema of [1]) that "x is preferred to y *and* y is preferred to z *and* z is preferred to x".

arguments will have different *resonances* for different audiences. To calculate this each agent has a *desideratabase* comprising a set of propositional formulae *desiderata* which the agent wishes to be satisfied, and a *weighting* which can be seen as a ranking over the set of possible worlds representing the set of classical interpretations given by the propositional language of the desiderata. Now an argument will have resonance for an agent if one or more desiderata (or their negations) can be derived from the support, and the weighting will determine the degree of resonance. Hunter uses this, together with the propositional cost of an argument (lengthier arguments are more expensive) to determine the *impact* of an argument.

Hunter's notion of arguments differs from ours in that we ascribe values to arguments. One form of argument that would link arguments to values is that of [2]. In their approach an argument justifying an action instantiates the following argument scheme:

- in the current circumstances S
- performing action A
- will result in new circumstances R
- which include goal G
- which promotes value V

Viewed in Hunter's terms we can see G as a desideratum derivable from the support comprising the theory which states that R is a consequence of performing A in S. Additionally we now also have V, the reason why the goal is desired. Introducing V has two important effects:

(1) it can distinguish two different arguments when agents wish to bring about the same state of affairs *for different reasons*: for example, one may wish to restore fox hunting on the economic grounds of protecting livestock, or simply for the hedonistic pleasure the activity affords.
(2) it can relate two states of affairs in so far as they promote the same value: for example poverty can be alleviated either by distributing food, or by distributing money.

It is in this ability to relate desirable states of affairs that values show their worth. It means that we can work with a smaller number of values than desiderata, that we can add a desired state of affairs without need to extend our set of values, and most importantly that desiring one state of affairs means that, in order to be consistent, we must also and equally desire other states of affairs. This greatly simplifies the weighting, and additionally means that some weightings can be seen as inconsistent since they differentially weight states of affairs relating to the same value. This is essential if arguments attempting to change audience membership, which are often required for persuasion, are to be possible. We would therefore argue that the use of values significantly enhances Hunter's representation with respect to practical reasoning. Moreover by replacing his notion of a desideratabase with a set of values and their ranking we can still make use of his theoretical notion of resonance to assess the impact of an argument for an audience.

7.3 Concluding Remarks

In this paper we have put forward a framework in which practical reasoning - reasoning about what should be done in a given situation - may be addressed. The distinctive features of practical reasoning derive from the acceptability of such arguments depending on the importance given to the values and purposes advanced if the argument is accepted by the audience to whom it is addressed. Accordingly we have extended the Argumentation Framework of Dung by associating arguments with the values advanced by their acceptance. From this property we can derive preferences between arguments with respect to particular audiences, and thus account for disagreements between different audiences. We have explored the decision problems that arise in this extended framework, presented a number of complexity results relating to these decision problems, and discussed the relation of the extended framework to the underlying abstract framework.

Another important question concerns how priorities between values can be determined by an agent in the course of practical reasoning. We have presented a dialogue mechanism for determining these priorities, and shown it to be sound and complete.

Practical reasoning is a crucial activity for any intelligent agent, since it is through action that intelligence manifests itself. The importance of the topic merits more investigation than it has so far been given, and we believe that we have provided a rich framework in which further investigation can be carried out.

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